

Contents

<i>Preface</i>	<i>page</i>	<i>ix</i>
1	Introduction and summary	1
2	PDE systems, pfaffian systems and vector field systems	7
2.1	ODE systems, vector fields and 1-parameter groups	7
2.2	First order PDE systems in one dependent variable, pfaffian equations and contact transformations	16
2.3	Jet bundles and contact pfaffian systems	19
2.4	The theorem of Frobenius	24
2.5	Mayer's blowing-up method for proving the Frobenius theorem	30
3	Cartan's local existence theorem	35
3.1	Involutions and characters	38
3.2	From involutions to complete systems	43
3.3	How general is the general solution?	55
3.4	Cauchy characteristics	60
3.5	Maximal involutions and integrable vector field systems	67
4	Involutivity and the prolongation theorem	79
4.1	Independence condition and involutivity	81
4.2	Prolongations	94
4.3	Explanation of the prolongation theorem	107
5	Drach's classification, second order PDEs in one dependent variable, and Monge characteristics	114
5.1	The classification of Drach	115
5.2	Second order PDEs in one unknown and their singular vector fields	117
5.3	Monge characteristic subsystems	124
6	Integration of vector field systems \mathcal{V} satisfying $\dim \mathcal{V}' = \dim \mathcal{V} + 1$	126

6.1	Maximal involutions	126
6.2	Complete subsystems	131
6.3	The generalized contact bracket	135
6.4	Reduction to a canonical form and systems of contact coordinates	139
6.5	How to find all maximal complete subsystems of \mathcal{V}	146
6.6	Contact transformations and Lie pseudogroups	152
6.7	Explicitly integrable systems	157
7	Higher order contact transformations	163
7.1	Lie's rectification theorem for first order PDE systems in one dependent variable	164
7.2	Bäcklund's theorems	168
7.3	Contact prolongations of local diffeomorphisms	173
8	Local Lie groups	175
8.1	The parameter group and its structure constants	175
8.2	The left- and right-invariant parameter groups	179
8.3	Left- and right-invariant vector fields and their dual Maurer–Cartan forms	182
8.4	One-parameter subgroups and the exponential mapping	189
8.5	The first and second fundamental theorems	201
8.6	The third fundamental theorem	207
8.7	Local transformation groups	213
9	Structural classification of 3-dimensional Lie algebras over the complex numbers	221
9.1	The classification	224
9.2	Realizations as transformation groups	229
10	Lie equations and Lie vector field systems	235
10.1	Characterization of ODE systems with fundamental solutions	242
10.2	Lie vector field systems associated to Lie groups	252
11	Second order PDEs in one dependent and two independent variables	270
11.1	Second order PDEs and associated vector field systems	270
11.2	Monge systems	277
11.3	A connection with line geometry	282
11.4	Darboux's method for hyperbolic PDEs	289
12	Hyperbolic PDEs with Monge systems admitting two or three first integrals	296

	12.1 First integrals of the first order	297
	12.2 Two first integrals for each Monge system	304
	12.3 How to find integral manifolds	312
	12.4 Integrable systems	317
	12.5 Two first integrals for one Monge system and three for the other	327
	12.6 Three first integrals for each Monge system	341
13	Classification of hyperbolic Goursat equations	346
	13.1 Goursat equations which are associated to 2-dimensional Lie groups	350
	13.2 Goursat equations associated to the projective group in one variable	356
14	Cartan's theory of Lie pseudogroups	368
	14.1 The first fundamental theorem	372
	14.2 The second fundamental theorem	383
	14.3 The third fundamental theorem	391
	14.4 The stability group and its Lie algebra	395
	14.5 Getting rid of inessential invariants	404
	14.6 Normal prolongations	409
15	The equivalence problem	421
	15.1 The simplest case: e -structures	426
	15.2 The general equivalence problem of Cartan	429
	15.3 Examples	433
16	Parabolic PDEs and associated PDE systems	442
	16.1 Parabolic PDEs for which the Monge system admits at least two first integrals	444
	16.2 Pfaffian systems of three and two dimensions in five variables	458
	16.3 Systems of two PDEs having a Cauchy characteristic vector field	464
17	The equivalence problem for general 3-dimensional pfaffian systems in five variables	471
	17.1 The naturally associated homogeneous polynomial \mathcal{F} of degree four in two variables	474
	17.2 \mathcal{F} has four simple roots	484
	17.3 \mathcal{F} vanishes identically	487
	17.4 \mathcal{F} has a root of multiplicity 4	492
	17.5 \mathcal{F} has one triple and one simple root	499
	17.6 \mathcal{F} has two double roots	500

18	Involutive second order PDE systems in one dependent and three independent variables, solved by the method of Monge	505
18.1	Preliminaries	510
18.2	Structural classification	515
18.3	A single PDE	527
18.4	Two PDEs	536
18.5	Three PDEs	549
18.6	Four and five PDEs	557
18.7	How to go further?	560
	<i>Bibliography</i>	565
	<i>Index</i>	570