

## CONTENTS

Preface . . . . .	V
Contents . . . . .	VII
List of examples . . . . .	XI
List of FORTRAN programs . . . . .	XV
List of frequently used symbols . . . . .	XVII
<b>1. Introduction . . . . .</b>	<b>1</b>
<b>2. Probabilities . . . . .</b>	<b>4</b>
2-1. Experiments, events, sample space . . . . .	4
2-2. The concept of probability . . . . .	5
2-3. Rules of probability calculus; conditional probability . . .	7
Exercises . . . . .	9
<b>3. Random variables; distributions of a random variable . . . . .</b>	<b>10</b>
3-1. Random variables . . . . .	10
3-2. Distributions of one random variable. . . . .	10
3-3. Functions of one random variable, expectation value, variance, moments . . . . .	12
3-4. Chebychev's inequality . . . . .	18
Exercises . . . . .	19
<b>4. Distributions of several random variables . . . . .</b>	<b>20</b>
4-1. Distribution function and probability density of two variables; conditional probability . . . . .	20
4-2. Expectation values, variances, covariances and correlation coefficient. . . . .	22
4-3. More than two variables; vector and matrix notation. . . . .	25
4-4. Transformation of variables . . . . .	28
4-5. Linear and orthogonal transformations; propagation of errors Exercises . . . . .	32 37
<b>5. Some important distributions and theorems . . . . .</b>	<b>39</b>
5-1. Binomial and multinomial distributions . . . . .	39

5-2.	Frequency; the law of large numbers . . . . .	42
5-3.	Hypergeometric distribution . . . . .	43
5-4.	Poisson distribution . . . . .	47
5-5.	Uniform distribution and an application: the Monte Carlo method . . . . .	52
5-5.1.	Probability density, expectation value, variance . . . . .	52
5-5.2.	Generation of uniformly distributed random numbers by computers . . . . .	53
5-5.3.	Generation of any distribution by transformation of the uniform distribution . . . . .	54
5-6.	The characteristic function of a distribution . . . . .	62
5-7.	The Laplace model of errors . . . . .	64
5-8.	Normal distribution. . . . .	67
5-9.	Quantitative properties of the normal distribution. . . . .	69
5-10.	Multivariate normal distribution . . . . .	72
5-11.	The central limit theorem . . . . .	78
5-12.	Experimental errors and normal distribution; Herschel's model . . . . .	79
5-13.	Convolution of distributions . . . . .	82
5-13.1.	Folding integrals. . . . .	82
5-13.2.	Convolution with the normal distribution . . . . .	85
	Exercises. . . . .	89
<b>6.</b>	<b>Sampling . . . . .</b>	<b>92</b>
6-1.	Random sampling; distribution of a sample; estimates . . . . .	92
6-2.	Sampling from continuous populations . . . . .	95
6-3.	Sampling from partitioned distributions. . . . .	97
6-4.	Sampling without replacement from finite populations; mean square deviation; degrees of freedom . . . . .	102
6-5.	Sampling from normal distributions; $\chi^2$ -distribution . . . . .	107
6-6.	$\chi^2$ and empirical variance . . . . .	111
6-7.	Sampling by counting. Small samples. . . . .	113
6-8.	Numerical and graphical analysis of sampled data with computer programs . . . . .	118
6-8.1.	Scatter diagram and histogram of a one-dimensional sample . . . . .	118
6-8.2.	Scatter diagram of a two-dimensional sample . . . . .	125
	Exercises . . . . .	131
<b>7.</b>	<b>The method of "maximum likelihood". . . . .</b>	<b>133</b>
7-1.	Likelihood quotient; likelihood function . . . . .	133
7-2.	The concept of maximum likelihood . . . . .	135
7-3.	Information inequality; minimum variance and sufficient estimates . . . . .	138
7-4.	Asymptotic properties of the likelihood function and of maximum likelihood estimators . . . . .	145

7-5. Solution of the likelihood equation by iteration . . . . .	147
7-6. Simultaneous estimation of several parameters. . . . .	148
7-7. Uniqueness of the method; confidence interval. . . . .	152
7-8. Bartlett's <i>S</i> -function . . . . .	154
Exercises . . . . .	157
<b>8. Testing of statistical hypotheses . . . . .</b>	<b>158</b>
8-1. <i>F</i> -test on equality of variances . . . . .	160
8-2. Student's test; comparison of means . . . . .	164
8-3. Some aspects of a general theory of tests . . . . .	169
8-4. Neyman-Pearson theorem and applications . . . . .	174
8-5. The likelihood ratio method. . . . .	177
8-6. The $\chi^2$ -test on goodness of fit . . . . .	182
Exercises . . . . .	189
<b>9. The method of least squares . . . . .</b>	<b>191</b>
9-1. Direct measurements with equal or different accuracy. . . . .	191
9-2. Indirect measurements . . . . .	196
9-2.1. The linear case. . . . .	196
9-2.2. The non-linear case. . . . .	204
9-2.3. Properties of the least squares solution; $\chi^2$ -test. . . . .	210
9-3. Constrained measurements . . . . .	213
9-3.1. The method of elements. . . . .	214
9-3.2. The method of Lagrangian multipliers . . . . .	216
9-4. The general case of least squares fitting . . . . .	221
9-5. A FORTRAN program for general least squares fitting; examples . . . . .	224
Exercises . . . . .	239
<b>10. Some remarks on minimization . . . . .</b>	<b>242</b>
10-1. Parameter estimation and minimization . . . . .	242
10-2. Different minimization procedures. . . . .	243
<b>11. Analysis of variance . . . . .</b>	<b>250</b>
11-1. One-way classification . . . . .	250
11-2. Some aspects of two-way classification. . . . .	255
11-3. A FORTRAN program for two-way classification. . . . .	264
Exercises. . . . .	269
<b>12. Linear regression . . . . .</b>	<b>270</b>
12-1. Linear regression as a simple case of least squares. . . . .	270
12-2. Confidence intervals. . . . .	274
12-3. Testing of hypotheses . . . . .	275
12-4. Linear regression and analysis of variance . . . . .	276

12-5. A general FORTRAN program for linear regression . . . . .	277
12-6. Interpretation of results from linear regression . . . . .	284
Exercises . . . . .	289
<b>13. Time series analysis . . . . .</b>	<b>291</b>
13-1. Time series. Trend . . . . .	291
13-2. Moving averages . . . . .	292
13-3. End effects . . . . .	296
13-4. Confidence interval . . . . .	296
13-5. A FORTRAN program for time series analysis . . . . .	298
13-6. A word of caution . . . . .	301
Exercise . . . . .	305
<b>Solution and discussion of the exercises . . . . .</b>	<b>306</b>

### APPENDICES

<b>A. Some elements of the FORTRAN programming language . . . . .</b>	<b>329</b>
<b>B. Short review of matrix calculus . . . . .</b>	<b>340</b>
B-1. Definitions of matrices and vectors . . . . .	340
B-2. Equality, addition, subtraction and multiplication of matrices	343
B-3. Determinant and inverse of a square matrix; solution of matrix	
equations . . . . .	347
B-4. FORTRAN programs for matrix handling . . . . .	354
<b>C. Elements of combinatorial analysis . . . . .</b>	<b>364</b>
<b>D. Euler's gamma-function . . . . .</b>	<b>367</b>
<b>E. Collection of important formulae . . . . .</b>	<b>369</b>
<b>F. Statistical tables . . . . .</b>	<b>389</b>
F-1. Poisson distribution . . . . .	389
F-2. Normal distribution function . . . . .	392
F-3. Fractiles of the normal distribution . . . . .	395
F-4. $\chi^2$ -distribution function . . . . .	398
F-5. Fractiles of $\chi^2$ -distribution . . . . .	400
F-6. $F$ -test . . . . .	401
F-7. Fractiles for Student's test . . . . .	406
F-8. Random numbers . . . . .	407
References and bibliography . . . . .	408
Subject index . . . . .	411
Index to FORTRAN statements and FORTRAN programs used in this book . . . . .	415