

CONTENTS

Preface		xi
1. Sets, Mappings and Transformations		
1.1	Introduction	1
1.2	Sets	1
1.3	Venn diagrams	4
1.4	Mappings	5
1.5	Linear transformations and matrices	8
1.6	Occurrence and uses of matrices	12
1.7	Operations with sets	14
1.8	Set algebra	19
1.9	Some elementary applications of set theory	20
	Problems 1	23
2. Matrix Algebra		
2.1	Laws of matrix algebra	25
2.2	Partitioning of matrices	28
2.3	Some special types of matrices	31
	Problems 2	40
3. The Inverse and Related Matrices		
3.1	Introduction	43
3.2	The adjoint matrix	44
3.3	The inverse matrix	46
3.4	Some properties of the inverse matrix	47
3.5	Evaluation of the inverse matrix by partitioning	48
3.6	Orthogonal matrices and orthogonal transformations	51
3.7	Unitary matrices	56
	Problems 3	58
4. Systems of Linear Algebraic Equations		
4.1	Introduction	62
4.2	Non-homogeneous equations	62
4.3	Homogeneous equations	67
4.4	Ill-conditioned equations	68
	Problems 4	69

Contents

5. Eigenvalues and Eigenvectors

5.1	Introduction	73
5.2	Eigenvalues and eigenvectors	74
5.3	Some properties of eigenvalues	78
5.4	Repeated eigenvalues	79
5.5	Orthogonal properties of eigenvectors	82
5.6	Real symmetric matrices	84
5.7	Hermitian matrices	87
5.8	Non-homogeneous equations	88
	Problems 5	90

6. Diagonalisation of Matrices

6.1	Introduction	92
6.2	Similar matrices	96
6.3	Diagonalisation of a matrix whose eigenvalues are all different	97
6.4	Matrices with repeated eigenvalues	100
6.5	Diagonalisation of real symmetric matrices	100
6.6	Diagonalisation of Hermitian matrices	102
6.7	Bilinear and quadratic forms	103
6.8	Lagrange's reduction of a quadratic form	105
6.9	Matrix diagonalisation of a real quadratic form	106
6.10	Hermitian forms	109
6.11	Simultaneous diagonalisation of two quadratic forms	110
	Problems 6	111

7. Functions of Matrices

7.1	Introduction	114
7.2	Cayley-Hamilton theorem	114
7.3	Powers of matrices	119
7.4	Some matrix series	121
7.5	Differentiation and integration of matrices	128
	Problems 7	131

8. Group Theory

8.1	Introduction	136
8.2	Group axioms	136
8.3	Examples of groups	138
8.4	Cyclic groups	141

Contents

8.5	Group tables	142
8.6	Isomorphic groups	145
8.7	Permutations: the symmetric group	146
8.8	Cayley's theorem	149
8.9	Subgroups and cosets	150
8.10	Some remarks on representations	151
	Problems 8	153
	Further Reading	155
	Answers to Problems	157
	Index	163