

Contents

Volume I

1	From Riemann Manifolds to Riemann Manifolds	1
	Mapping from a Left Two-Dimensional Riemann Manifold to a Right Two-Dimensional Riemann Manifold	1
1-1	Cauchy–Green Deformation Tensor	6
1-2	Stretch or Length Distortion	12
1-3	Two Examples: Pseudo-Cylindrical and Orthogonal Map Projections	21
1-4	Euler–Lagrange Deformation Tensor	33
1-5	One Example: Orthogonal Map Projection	38
1-6	Review: The Deformation Measures	42
1-7	Angular Shear	42
1-8	Relative Angular Shear	45
1-9	Equivalence Theorem of Conformal Mapping	48
1-10	Two Examples: Mercator Projection and Stereographic Projection	60
1-11	Areal Distortion	83
1-12	Equivalence Theorem of Equiareal Mapping	86
1-13	One Example: Mapping from an Ellipsoid-of-Revolution to the Sphere	88
1-14	Review: The Canonical Criteria	92
	1-141 Isometry	92
	1-142 Equidistant Mapping of Submanifolds	94
	1-143 Canonical Criteria	95
	1-144 Optimal Map Projections	99
	1-145 Maximal Angular Distortion	100
1-15	Exercise: The Armadillo Double Projection	106
2	From Riemann Manifolds to Euclidean Manifolds	111
	Mapping from a Left Two-Dimensional Riemann Manifold to a Right Two-Dimensional Euclidean Manifold	111
2-1	Eigenspace Analysis, Cauchy–Green Deformation Tensor	111
2-2	Eigenspace Analysis, Euler–Lagrange Deformation Tensor	114
2-3	The Equivalence Theorem for Conformal Mappings	116
	2-31 Conformeomorphism	116
	2-32 Higher-Dimensional Conformal Mapping	117

2-4	The Equivalence Theorem for Equiareal Mappings	122
2-5	Canonical Criteria for Conformal, Equiareal, and Other Mappings	126
2-6	Polar Decomposition and Simultaneous Diagonalization of Three Matrices	127
3	Coordinates	129
	Coordinates (direct, transverse, oblique aspects)	129
3-1	Coordinates Relating to Manifolds	130
3-2	Killing Vectors of Symmetry	139
3-3	The Oblique Frame of Reference of the Sphere	144
3-31	A First Design of an Oblique Frame of Reference of the Sphere	144
3-32	A Second Design of an Oblique Frame of Reference of the Sphere	152
3-33	The Transverse Frame of Reference of the Sphere: Part One	156
3-34	The Transverse Frame of Reference of the Sphere: Part Two	158
3-35	Transformations Between Oblique Frames of Reference: First Design, Second Design	159
3-36	Numerical Examples	162
3-4	The Oblique Frame of Reference of the Ellipsoid-of-Revolution	163
3-41	The Direct and Inverse Transformations of the Normal Frame to the Oblique Frame	164
3-42	The Intersection of the Ellipsoid-of-Revolution and the Central Oblique Plane	165
3-43	The Oblique Quasi-Spherical Coordinates	166
3-44	The Arc Length of the Oblique Equator in Oblique Quasi-Spherical Coordinates	167
3-45	Direct Transformation of Oblique Quasi-Spherical Longitude/Latitude	170
3-46	Inverse Transformation of Oblique Quasi-Spherical Longitude/Latitude	173
4	Surfaces of Gaussian Curvature Zero	177
	Classification of Surfaces of Gaussian Curvature Zero in a Two-Dimensional Euclidean Space	177
4-1	Ruled Surfaces	177
4-2	Developable Surfaces	181
5	“Sphere to Tangential Plane”: Polar (Normal) Aspect	185
	Mapping the Sphere to a Tangential Plane: Polar (Normal) Aspect	185
5-1	General Mapping Equations	188
5-2	Special Mapping Equations	191
5-21	Equidistant Mapping (Postel Projection)	191
5-22	Conformal Mapping (Stereographic Projection, UPS)	193
5-23	Equiareal Mapping (Lambert Projection)	197
5-24	Normal Perspective Mappings	200
5-25	What Are the Best Polar Azimuthal Projections of “Sphere to Plane”?	224

5-3	The Pseudo-Azimuthal Projection	230
5-4	The Wiechel Polar Pseudo-Azimuthal Projection	235
6	“Sphere to Tangential Plane”: Transverse Aspect	239
	Mapping the Sphere to a Tangential Plane: Meta-azimuthal Projections in the Transverse Aspect	239
6-1	General Mapping Equations	239
6-2	Special Mapping Equations	240
6-21	Equidistant Mapping (Transverse Postel Projection)	241
6-22	Conformal Mapping (Transverse Stereographic Projection, Transverse UPS)	242
6-23	Equal Area Mapping (Transverse Lambert Projection)	243
7	“Sphere to Tangential Plane”: Oblique Aspect	247
	Mapping the Sphere to a Tangential Plane: Meta-azimuthal Projections in the Oblique Aspect	247
7-1	General Mapping Equations	248
7-2	Special Mapping Equations	248
7-21	Equidistant Mapping (Oblique Postel Projection)	248
7-22	Conformal Mapping (Oblique Stereographic Projection, Oblique UPS)	250
7-23	Equal Area Mapping (Oblique Lambert Projection)	252
8	Ellipsoid-of-Revolution to Tangential Plane	255
	Mapping the Ellipsoid to a Tangential Plane (Azimuthal Projections in the Normal Aspect)	255
8-1	General Mapping Equations	257
8-2	Special Mapping Equations	260
8-21	Equidistant Mapping	260
8-22	Conformal Mapping	267
8-23	Equiareal Mapping	274
8-3	Perspective Mapping Equations	277
8-31	The First Derivation	279
8-32	The Special Case “Sphere to Tangential Plane”	287
8-33	An Alternative Approach for a Topographic Point	288
9	Ellipsoid-of-Revolution to Sphere and from Sphere to Plane	293
	Mapping the Ellipsoid to Sphere and from Sphere to Plane (Double Projection, “Authalic” Projection)	293
9-1	General Mapping Equations “Ellipsoid-of-Revolution to Plane”	293
9-11	The Setup of the Mapping Equations “Ellipsoid-of-Revolution to Plane”	294
9-12	The Metric Tensor of the Ellipsoid-of-Revolution, the First Differential Form	294

9-13	The Curvature Tensor of the Ellipsoid-of-Revolution, the Second Differential Form	295
9-14	The Metric Tensor of the Sphere, the First Differential Form.....	297
9-15	The Curvature Tensor of the Sphere, the Second Differential Form.....	298
9-16	Deformation of the First Kind	298
9-17	Deformation of the Second Kind	301
9-2	The Conformal Mappings “Ellipsoid-of-Revolution to Plane”	302
9-3	The Equal Area Mappings “Ellipsoid-of-Revolution to Plane”	308
10	“Sphere to Cylinder”: Polar Aspect	311
	Mapping the Sphere to a Cylinder: Polar Aspect	311
10-1	General Mapping Equations	311
10-2	Special Mapping Equations	314
10-21	Equidistant Mapping (Plate Carrée Projection)	315
10-22	Conformal Mapping (Mercator Projection)	316
10-23	Equal Area Mapping (Lambert Projection)	317
10-3	Optimal Cylinder Projections	318
11	“Sphere to Cylinder”: Transverse Aspect	325
	Mapping the Sphere to a Cylinder: Meta-cylindrical Projections in the Transverse Aspect	325
11-1	General Mapping Equations	325
11-2	Special Mapping Equations	327
11-21	Equidistant Mapping (Transverse Plate Carrée Projection)	327
11-22	Conformal Mapping (Transverse Mercator Projection)	328
11-23	Equal Area Mapping (Transverse Lambert Projection)	328
12	“Sphere to Cylinder”: Oblique Aspect	331
	Mapping the Sphere to a Cylinder: Meta-cylindrical Projections in the Oblique Aspect	331
12-1	General Mapping Equations	331
12-2	Special Mapping Equations	333
12-21	Equidistant Mapping (Oblique Plate Carrée Projection).....	333
12-22	Conformal Mapping (Oblique Mercator Projection).....	333
12-23	Equal Area Mapping (Oblique Lambert Projection)	335
13	“Sphere to Cylinder”: Pseudo-Cylindrical Projections	337
	Mapping the Sphere to a Cylinder: Pseudo-Cylindrical Projections	337
13-1	General Mapping Equations	337
13-2	Special Mapping Equations	339
13-21	Sinusoidal Pseudo-Cylindrical Mapping (J. Cossin, N. Sanson, J. Flamsteed)	340
13-22	Elliptic Pseudo-Cylindrical Mapping (C. B. Mollweide)	340
13-23	Parabolic Pseudo-Cylindrical Mapping (J. E. E. Craster).....	343
13-24	Rectilinear Pseudo-Cylindrical Mapping (Eckert II).....	344

14	“Ellipsoid-of-Revolution to Cylinder”: Polar Aspect	347
	Mapping the Ellipsoid to a Cylinder (Polar Aspect, Generalization for Rotational-Symmetric Surfaces)	347
14-1	General Mapping Equations	347
14-2	Special Mapping Equations	348
14-21	Special Normal Cylindric Mapping (Equidistant: Meridian Circles, Conformal: Equator)	349
14-22	Special Normal Cylindric Mapping (Normal Conformal, Equidistant: Equator)	350
14-23	Special Normal Cylindric Mapping (Normal Equiareal, Equidistant: Equator)	351
14-24	Summary (Cylindric Mapping Equations)	353
14-3	General Cylindric Mappings (Equidistant, Rotational-Symmetric Figure)	354
14-31	Special Normal Cylindric Mapping (Equidistant: Equator, Set of Meridian Circles)	356
14-32	Special Normal Conformal Cylindric Mapping (Equidistant: Equator) ...	356
14-33	Special Normal Equiareal Cylindric Mapping (Equidistant + Conformal: Equator)	356
14-34	An Example (Mapping the Torus).....	357
15	“Ellipsoid-of-Revolution to Cylinder”: Transverse Aspect	361
	Mapping the Ellipsoid to a Cylinder (Transverse Mercator and Gauss–Krueger Mappings)	361
15-1	The Equations Governing Conformal Mapping	365
15-2	A Fundamental Solution for the Korn–Lichtenstein Equations	368
15-3	Constraints to the Korn–Lichtenstein Equations (Gauss–Krueger/UTM Mappings)	375
15-4	Principal Distortions and Various Optimal Designs (UTM Mappings)	382
15-5	Examples (Gauss–Krueger/UTM Coordinates)	387
15-6	Strip Transformation of Conformal Coordinates (Gauss–Krueger/UTM Mappings)	400
15-61	Two-Step-Approach to Strip Transformations	401
15-62	Two Examples of Strip Transformations	410