

Contents

Preface	xiii
Notation and Acronyms	xvii
1 Exchangeability and subjective probability	1
1.1 Introduction	1
1.2 Families of exchangeable events	6
1.2.1 Extendibility and de Finetti's theorem	12
1.2.2 The problem of prediction	15
1.2.3 More on infinitely extendible families	18
1.3 Exchangeable random quantities	21
1.3.1 Extendibility and de Finetti's theorem for exchangeable random variables	23
1.3.2 The problem of prediction	29
1.4 de Finetti-type theorems and parametric models	32
1.4.1 Parametric models and prediction sufficiency	35
1.5 Exercises	39
1.6 Bibliography	42
2 Exchangeable lifetimes	47
2.1 Introduction	47
2.2 Positive exchangeable random quantities	52
2.3 Multivariate conditional hazard rates	70
2.4 Further aspects of m.c.h.r.	80
2.4.1 On the use of the m.c.h.r. functions	80
2.4.2 Dynamic histories, total time on test statistic and total hazard transform	83
2.4.3 M.c.h.r. functions and dynamic sufficiency	90
2.5 Exercises	92
2.6 Bibliography	95

3	Some concepts of dependence and aging	99
3.1	Introduction	99
3.1.1	One-dimensional stochastic orderings	101
3.1.2	Stochastic monotonicity and orderings for conditional distributions	105
3.2	Multivariate stochastic orderings	107
3.2.1	Usual multivariate stochastic ordering	107
3.2.2	Multivariate likelihood ratio ordering	109
3.2.3	Multivariate hazard rate and cumulative hazard rate orderings	110
3.2.4	Some properties of multivariate stochastic orderings and examples	111
3.3	Some notions of dependence	115
3.3.1	Positive dependence	115
3.3.2	Negative dependence	120
3.3.3	Simpson-type paradoxes and aspects of dependence in Bayesian analysis	122
3.3.4	Likelihood-ratio comparisons between posterior distributions	124
3.4	Some notions of aging	128
3.4.1	One-dimensional notions of aging	128
3.4.2	Dynamic multivariate notions of aging	134
3.4.3	The case of exchangeable lifetimes	136
3.5	Exercises	140
3.6	Bibliography	143
4	Bayesian models of aging	147
4.1	Introduction	147
4.2	Schur survival functions	151
4.2.1	Basic background about majorization	152
4.2.2	Schur properties of survival functions and multivariate aging	156
4.2.3	Examples of Schur survival functions	160
4.2.4	Schur survival functions and dependence	163
4.3	Schur density functions	164
4.3.1	Schur-constant densities	166
4.3.2	Examples of Schur densities	169
4.3.3	Properties of Schur densities	172
4.4	Further aspects of Bayesian aging	176
4.4.1	Schur densities and TTT plots	176
4.4.2	Some other notions of Bayesian aging	179
4.4.3	Heterogeneity and multivariate negative aging	180
4.4.4	A few bibliographical remarks	183

4.4.5	Extensions to non-exchangeable cases	183
4.5	Exercises	184
4.6	Bibliography	187
5	Bayesian decisions, orderings, and majorization	191
5.1	Introduction	191
5.1.1	Statistical decision problems	196
5.1.2	Statistical decision problems and sufficiency	203
5.1.3	Some technical aspects	204
5.2	Stochastic orderings and orderings of decisions	206
5.3	Orderings of residual lifetimes and majorization	212
5.3.1	The case of observations containing failure data	218
5.4	Burn-in problems for exchangeable lifetimes	224
5.4.1	The case of i.i.d. lifetimes	225
5.4.2	Dependence and optimal adaptive burn-in procedures	228
5.4.3	Burn-in, optimal stopping, monotonicity, and Markovianity	231
5.4.4	Stochastic orderings and open-loop optimal adaptive burn-in procedures	233
5.5	Exercises	240
5.6	Bibliography	241
	Essential bibliography	245
	Index	247