

Contents

Preface to the first edition	xi
Preface to the second and third editions	xvii
1 Introduction	1
1.1 What is analysis?	1
1.2 Why do analysis?	2
2 Starting at the beginning: the natural numbers	13
2.1 The Peano axioms	15
2.2 Addition	24
2.3 Multiplication	29
3 Set theory	33
3.1 Fundamentals	33
3.2 Russell's paradox (Optional)	46
3.3 Functions	49
3.4 Images and inverse images	56
3.5 Cartesian products	62
3.6 Cardinality of sets	67
4 Integers and rationals	74
4.1 The integers	74
4.2 The rationals	81
4.3 Absolute value and exponentiation	86
4.4 Gaps in the rational numbers	90
5 The real numbers	94
5.1 Cauchy sequences	96
5.2 Equivalent Cauchy sequences	100
5.3 The construction of the real numbers	102
5.4 Ordering the reals	111

5.5	The least upper bound property	116
5.6	Real exponentiation, part I	121
6	Limits of sequences	126
6.1	Convergence and limit laws	126
6.2	The Extended real number system	133
6.3	Suprema and Infima of sequences	137
6.4	Limsup, Liminf, and limit points	139
6.5	Some standard limits	148
6.6	Subsequences	149
6.7	Real exponentiation, part II	152
7	Series	155
7.1	Finite series	155
7.2	Infinite series	164
7.3	Sums of non-negative numbers	170
7.4	Rearrangement of series	174
7.5	The root and ratio tests	178
8	Infinite sets	181
8.1	Countability	181
8.2	Summation on infinite sets	188
8.3	Uncountable sets	195
8.4	The axiom of choice	198
8.5	Ordered sets	202
9	Continuous functions on $\mathbb{R}$	211
9.1	Subsets of the real line	211
9.2	The algebra of real-valued functions	217
9.3	Limiting values of functions	220
9.4	Continuous functions	227
9.5	Left and right limits	231
9.6	The maximum principle	234
9.7	The intermediate value theorem	238
9.8	Monotonic functions	241
9.9	Uniform continuity	243
9.10	Limits at infinity	249
10	Differentiation of functions	251
10.1	Basic definitions	251

10.2	Local maxima, local minima, and derivatives	257
10.3	Monotone functions and derivatives	260
10.4	Inverse functions and derivatives	261
10.5	L'Hôpital's rule	264
11	The Riemann integral	267
11.1	Partitions	268
11.2	Piecewise constant functions	272
11.3	Upper and lower Riemann integrals	276
11.4	Basic properties of the Riemann integral	280
11.5	Riemann integrability of continuous functions	285
11.6	Riemann integrability of monotone functions	289
11.7	A non-Riemann integrable function	291
11.8	The Riemann-Stieltjes integral	292
11.9	The two fundamental theorems of calculus	295
11.10	Consequences of the fundamental theorems	300
A	Appendix: the basics of mathematical logic	305
A.1	Mathematical statements	306
A.2	Implication	312
A.3	The structure of proofs	317
A.4	Variables and quantifiers	320
A.5	Nested quantifiers	324
A.6	Some examples of proofs and quantifiers	327
A.7	Equality	329
B	Appendix: the decimal system	331
B.1	The decimal representation of natural numbers	332
B.2	The decimal representation of real numbers	335
	Index	339