

Contents

PREFACE

vii

I MATHEMATICAL PRELIMINARIES

| | |
|--|----|
| Euclidean Space | 1 |
| 1 Vectors and vector spaces | 1 |
| 2 Linear dependence | 3 |
| 3 Dimensions of a vector space, basis | 4 |
| 4 Euclidean space | 4 |
| 5 Norm of a vector | 6 |
| Linear Algebraic Equations | 8 |
| 6 General form | 8 |
| 7 Particular case: when A is nonsingular square matrix | 9 |
| 8 Consistent system of equations | 10 |
| 9 Linearly independent consistent equations | 11 |
| 10 Homogeneous equations | 13 |
| Convex Sets | 15 |
| 11 Open and closed sets in E_n | 15 |
| 12 Convex linear combinations, convex sets | 17 |
| 13 Intersection of convex sets, convex hull of a set | 20 |
| 14 Vertices or extreme points of a convex set | 22 |
| 15 Convex polyhedron | 23 |
| 16 Hyperplanes, half-spaces and polytopes | 24 |
| 17 Separating and supporting hyperplanes | 26 |
| 18 Vertices of a closed bounded convex set | 28 |
| 19 Summary | 31 |
| Quadratic Forms | 31 |
| 20 Quadratic forms | 31 |
| <i>Bibliographical note</i> | 34 |
| <i>Problems I</i> | 34 |

II EXTREMA OF FUNCTIONS

| | | |
|----|---|----|
| 1 | Real-valued function | 40 |
| 2 | Partial derivatives, gradient vector | 40 |
| 3 | Taylor series | 41 |
| 4 | Directional derivative, direction of steepest descent | 41 |
| 5 | Local and global extrema | 42 |
| 6 | Limitations of the method of differential calculus | 43 |
| 7 | Unconstrained extrema of differentiable functions | 44 |
| 8 | Constrained extrema | 46 |
| 9 | Implicit function theorem | 47 |
| 10 | Method of Lagrange multipliers | 48 |
| 11 | Convex functions | 50 |
| 12 | General problem of mathematical programming | 54 |
| | <i>Bibliographical note</i> | 55 |
| | <i>Problems II</i> | 56 |

III LINEAR PROGRAMMING

| | | |
|----|--|----|
| 1 | Introduction | 58 |
| 2 | LP in two-dimensional space | 58 |
| 3 | General LP problem | 61 |
| 4 | Feasible solutions | 62 |
| 5 | Basic solutions | 63 |
| 6 | Basic feasible solutions | 64 |
| 7 | Optimal solutions | 66 |
| 8 | Summary | 68 |
| 9 | Simplex method | 69 |
| 10 | Canonical form of equations | 70 |
| 11 | Simplex method (numerical example) | 71 |
| 12 | Simplex tableau | 74 |
| 13 | Finding the first b.f.s., artificial variables | 75 |
| 14 | Degeneracy | 78 |
| 15 | Simplex multipliers | 79 |
| 16 | Revised simplex method | 80 |
| 17 | Duality in LP problems | 84 |
| 18 | Duality theorems | 86 |
| 19 | Sensitivity analysis and parametric LP | 89 |
| 20 | Integer programming | 92 |
| 21 | Further developments in LP | 93 |
| | <i>Historical note</i> | 93 |
| | <i>Bibliographical note</i> | 94 |

| | |
|---------------------|----|
| <i>Problems III</i> | 94 |
|---------------------|----|

IV TRANSPORTATION PROBLEM

| | | |
|----|------------------------------------|-----|
| 1 | Introduction | 97 |
| 2 | Transportation problem | 97 |
| 3 | Traingular basis | 98 |
| 4 | Finding a basic feasible solution | 100 |
| 5 | Testing for optimality | 102 |
| 6 | Changing the basis | 104 |
| 7 | Degeneracy | 105 |
| 8 | Unbalanced problem | 106 |
| 9 | Transportation with transshipment | 106 |
| 10 | Caterer problem | 109 |
| 11 | Assignment problem | 112 |
| 12 | Generalized transportation problem | 115 |
| | <i>Historical note</i> | 115 |
| | <i>Bibliographical note</i> | 116 |
| | <i>Problems IV</i> | 116 |

V FLOW AND POTENTIAL IN NETWORKS

| | | |
|---|---|-----|
| 1 | Introduction | 121 |
| 2 | Graphs: definitions and notation | 121 |
| 3 | Minimum path problem | 125 |
| 4 | Spanning tree of minimum length | 131 |
| 5 | Problem of minimum potential difference | 135 |
| 6 | Scheduling of sequential activities | 137 |
| 7 | Maximum flow problem | 140 |
| 8 | Duality in the maximum flow problem | 146 |
| 9 | Generalized problem of maximum flow | 147 |
| | <i>Historical note</i> | 150 |
| | <i>Bibliographical note</i> | 150 |
| | <i>Problems V</i> | 150 |

VI NONLINEAR CONVEX PROGRAMMING

| | | |
|---|-----------------------------------|-----|
| 1 | Introduction | 154 |
| 2 | Lagrangian function; saddle point | 154 |
| 3 | Kuhn-Tucker theory | 155 |
| 4 | Quadratic programming | 161 |
| | <i>Historical note</i> | 167 |

| | | |
|-----------------------------------|--|-----|
| | <i>Bibliographical note</i> | 167 |
| | <i>Problems VI</i> | 167 |
| VII DYNAMIC PROGRAMMING | | |
| 1 | Introduction | 169 |
| 2 | Problem I: a minimum path problem | 169 |
| 3 | Problem II: single additive constraint, additively separable return | 173 |
| 4 | Problem III: single multiplicative constraint, additively separable return | 177 |
| 5 | Problem IV: single additive constraint, multiplicatively separable return | 179 |
| 6 | Computational economy in DP | 180 |
| 7 | Serial multistage model | 181 |
| 8 | Examples of failure | 183 |
| 9 | Decomposition | 184 |
| 10 | Backward and forward recursion | 186 |
| 11 | Systems with more than one constraint | 189 |
| 12 | Application of DP to continuous systems | 191 |
| | <i>Bibliographical note</i> | 192 |
| | <i>Problems VII</i> | 193 |
| VIII GEOMETRIC PROGRAMMING | | |
| 1 | Introduction | 196 |
| 2 | Illustrative examples | 197 |
| 3 | General method | 203 |
| | <i>Historical note</i> | 204 |
| | <i>Bibliographical note</i> | 205 |
| | <i>Problems VIII</i> | 205 |
| IX THEORY OF GAMES | | |
| 1 | Introduction | 206 |
| 2 | Matrix (or rectangular) games | 207 |
| 3 | Problem of game theory | 208 |
| 4 | Minimax theorem, saddle point | 209 |
| 5 | Strategies and pay off | 213 |
| 6 | Theorems of matrix games | 214 |
| 7 | Graphical solution | 219 |
| 8 | Notion of dominance | 222 |

| | | |
|---|---|-----|
| 9 | Rectangular game as an LP problem | 222 |
| | <i>Historical note</i> | 224 |
| | <i>Bibliographical note</i> | 224 |
| | <i>Problems IX</i> | 224 |
| X DIRECT SEARCH AND GRADIENT METHODS | | |
| 1 | Introduction | 226 |
| | One-dimensional Search | 228 |
| 2 | Unimodal functions | 228 |
| 3 | Search plans (one variable) | 228 |
| 4 | Fibonacci search plan | 230 |
| 5 | Golden section plan | 233 |
| 6 | Rosenbrock method | 233 |
| 7 | Methods requiring $f(x)$ to be differentiable | 234 |
| | Multi-dimensional Search | 236 |
| 8 | n-dimensional problem | 236 |
| 9 | The basic step—search along a line | 237 |
| 10 | Basic methods of choosing a direction | 239 |
| 11 | Conjugate directions | 241 |
| 12 | Conjugate gradient method | 243 |
| 13 | PARTAN method | 244 |
| 14 | Scaling | 245 |
| 15 | Constrained problem, gradient projection | 245 |
| | <i>Historical note</i> | 249 |
| | <i>Bibliographical note</i> | 249 |
| | <i>Problems X</i> | 249 |
| | BIBLIOGRAPHY | 251 |
| | INDEX | 255 |