UNIVERSITÀ DI PISA
DIPARTIMENTO DI MATEMATICA

Dottorato di ricerca in matematica

THE BASIC THEORY OF ELLIPTIC SURFACES

Notes of lectures

by

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