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Numerical Analysis of Stochastic Functional Differential Equations

Longtime Asymptotics and Probabilistic
Characteristics

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