

461

ASTÉRIQUE

2025

ISOLATED QUOTIENT SINGULARITIES
IN POSITIVE CHARACTERISTIC

C. Liedtke, G. Martin & Y. Matsumoto

SOCIÉTÉ MATHÉMATIQUE DE FRANCE

CONTENTS

1. Introduction	1
1.1. Quotient singularities over the complex numbers	1
1.2. Properties of complex isolated quotient singularities	4
1.3. Local fundamental groups	5
1.4. Counterexamples in positive characteristic	6
1.5. Linearly reductive quotient singularities	7
1.6. Torsors over the rational double points in characteristic p	10
1.7. Outlook	15
2. Linearly Reductive Quotient Singularities	21
2.1. Introduction	21
2.2. Linearly reductive group schemes	27
2.3. Very small linearly reductive subgroup schemes of \mathbf{GL}_d	34
2.4. Generalities on F-singularities	40
2.5. Local fundamental groups and class groups	41
2.6. Quotient singularities and linearization	43
2.7. Properties of lrq singularities	46
2.8. Uniqueness of the quotient presentation	52
2.9. Rigidity of lrq singularities in dimension $d \geq 4$	56
2.10. Deformation theory of lrq threefolds	58
2.11. F-regular surfaces are lrq surfaces	65
2.12. Riemenschneider's conjecture for lrq singularities	68
3. Torsors over the Rational Double Points in Characteristic p	75
3.1. Introduction	75
3.2. F-regular and canonical singularities in dimension two	82
3.3. Finite group schemes	83
3.4. Maximal \mathcal{C} -subsheaves	90
3.5. Local torsors and how to detect them	94
3.6. Local torsors over the rational double points	106
3.7. Detecting smoothness via local torsors	115
3.8. Very small actions and quotient singularities	120
3.9. Quotient and non-quotient RDPs	126
3.10. Pathologies and counter-examples	144
Bibliography	153