

Contents

	Preface	xi
1	Introduction to Differential Equations	1
1.1	Introduction	1
1.2	Classification of Differential Equations and Direction Fields	11
1.3	An Excursion: Numerical Solutions of Differential Equations	20
	Review Exercises	26
2	First-Order Equations	28
2.1	Separation of Variables	28
2.2	Exact Equations and Integrating Factors	40
2.3	Linear First-Order Differential Equations	46
2.4	Homogeneous Equations and Substitution Techniques	54
2.5	Simple Electric Circuits	61
2.6	Further Applications: Compartmental Analysis and Curves of Pursuit	65
2.7	Successive Approximations: If Time Permits	74
2.8	An Excursion: First-Order Difference Equations	77
	Review Exercises	81
3	Second- and Higher-Order Linear Differential Equations	83
3.1	Theory of Linear Differential Equations	83
3.2	Using One Solution to Find Another: Reduction of Order	97
3.3	Homogeneous Equations with Constant Coefficients: Real Roots	100
3.4	Homogeneous Equations with Constant Coefficients: Complex Roots	103
3.5	Nonhomogeneous Equations: Undetermined Coefficients	105
3.6	Nonhomogeneous Equations: Variation of Parameters	112
3.7	Euler Equations	118
3.8	Higher-Order Linear Differential Equations	122
3.9	An Excursion: Second-Order Difference Equations	128
	Review Exercises	135
4	Applications of Linear Differential Equations	137
4.1	Vibrational Motion: Simple Harmonic Motion	137
4.2	Vibrational Motion: Damped Vibrations	143
4.3	Vibrational Motion: Forced Vibrations	146
4.4	Electric Circuits	152
	Review Exercises	155

5	Power Series Solutions of Differential Equations	157
5.1	The Power Series Method	157
5.2	Ordinary and Singular Points	166
5.3	The Method of Frobenius: The Indicial Equation	173
5.4	Bessel Functions	186
5.5	Legendre Polynomials	195
5.6	An Excursion: Oscillatory Solutions of Linear Second-Order Differential Equations with Variable Coefficients	201
	Review Exercises	207
6	Laplace Transforms	209
6.1	Introduction: Definition and Basic Properties of the Laplace Transform	209
6.2	Solving Initial-Value Problems by Laplace Transform Methods	221
6.3	Step Functions, Impulse Functions, and Periodic Functions	234
6.4	Some Differential Equations with Discontinuous Forcing Functions: Applications to Electric Circuits (Optional)	247
6.5	The Transform of Convolution Integrals	252
	Review Exercises	258
7	Systems of Linear Differential Equations	259
7.1	Systems of First-Order Equations	260
7.2	Linear Systems: Theory	266
	Nonmatrix Methods	271
7.3	The Method of Elimination and the Method of Determinants for Linear Systems with Constant Coefficients	271
7.4	Laplace Transform Methods for Systems	280
	Matrix Methods	285
7.5	Review of Matrices	285
7.6	Matrices and Linear Systems of Differential Equations	299
7.7	Fundamental and Principal Matrix Solutions of a Homogeneous System of Differential Equations	304
7.8	Eigenvalues and Eigenvectors	315
7.9	Computation of the Principal Matrix Solution	327
7.10	Nonhomogeneous Systems	337
7.11	An Excursion: The Matrix Exponential	344
	Review Exercises	352
8	Applications of Systems of Differential Equations	354
8.1	Electric Circuits with Several Loops	354
8.2	Chemical Mixture and Population Biology Problems	360
8.3	Mechanical Systems	366
8.4	An Excursion: A Model for Epidemics	369
	Review Exercises	374
9	Numerical Methods	376
9.1	Error Analysis	376
9.2	Euler Methods for First-Order Differential Equations	379
9.3	The Runge-Kutta Method	386
9.4	Predictor-Corrector Formulas	393

9.5	Error Analysis for the Euler Method: If Time Permits	398
9.6	An Excursion: Numerical Instability Caused by the Propagation of Round-Off Error	402
9.7	Numerical Solution of Systems and Boundary Value Problems	404
	Review Exercises	410
10	Nonlinear Equations and Stability	411
10.1	Introduction	411
10.2	Critical Points, Stability, and Phase Portraits for Linear Systems	419
10.3	Stability of Nonlinear Systems	433
10.4	An Excursion: Lyapunov's Method	443
	Review Exercises	450
11	Fourier Series and Boundary Value Problems	451
11.1	Introduction to Trigonometric Series	451
11.2	Orthogonal Sets of Functions	458
11.3	Fourier Series	463
11.4	Half-Range Expansions	473
11.5	Sturm-Liouville Problems	479
11.6	The Orthogonality Theorem	484
11.7	Nonhomogeneous Boundary Value Problems	488
11.8	An Excursion: Least Squares Polynomial Approximation	494
	Review Exercises	497
12	Partial Differential Equations	499
12.1	First-Order Linear Partial Differential Equations	499
12.2	Initial-Value Problems for Quasi-linear First-Order Equations	506
12.3	Applications to the Theory of Shocks in Gas Dynamics: If Time Permits	511
12.4	Classification of Linear Second-Order Equations	515
12.5	The Vibrating String: d'Alembert's Method	521
12.6	Separation of Variables: The Wave Equation	527
12.7	Heat Flow and the Heat Equation	532
12.8	Two-Dimensional Heat Flow and Laplace's Equation	536
12.9	Laplace's Equation in Polar and Spherical Coordinates	540
12.10	Laplace Transform Methods for Partial Differential Equations	549
	Review Exercises	554
	Appendixes	
1	Integral Tables	A1
2	Table of Laplace Transforms	A11
3	The Existence and Uniqueness of Solutions	A14
4	Determinants	A29
5	Complex Numbers	A39
	Answers to Odd-Numbered Problems	A47
	Index	I1