

Contents

PREFACE

vii

PART I

Chapter 1. Geometry of Banach spaces

10. Introduction	3
Summary of the chapter Terminology and notation Continuous selections	
11. Angles, splittings, and dihedra	7
Angular distance and related concepts Splittings Dihedra	
12. Coupled spaces	13
Coupled spaces Subspaces Selections and splittings Reflexive spaces	
13. The class of subspaces of a Banach space	18
Two metrics The complemented subspaces The class of closed dihedra A lemma on continuously varying subspaces	
14. Hilbert space	26
Notation Angles, splittings, and dihedra The set of subspaces	
15. Notes to Chapter 1	31

Chapter 2. Function spaces

20. Introduction	33
Summary of the chapter Further terminology and notation for abstract spaces. The relations “stronger than” and \leq Functions and function spaces The Lebesgue spaces $L^p(X)$ The space $L(X)$ $J = R$ and $J = R_+$ Translation operators	
21. \mathcal{N} -spaces	41
The lattice $\mathcal{N}(X)$ Banach spaces in $\mathcal{N}(X)$ Local closure Completion The relation \Rightarrow	

22.	\mathcal{F} -spaces	46
	The lattices \mathcal{F} and $b\mathcal{F}$. Local closure The operators k_0, k, f . Lean and full spaces The class \mathcal{F}_K Associate spaces Thin spaces The domains R, R_-, R_+ . Cutting and splicing at 0 The class $\mathcal{F}(X)$	
23.	\mathcal{T} -spaces	57
	The classes $\mathcal{T}, \mathcal{T}^-$ The operator T^- The class \mathcal{T}_K Associate spaces in \mathcal{T} The functions $\alpha(\mathbf{F}; l), \beta(\mathbf{F}; l)$; the spaces $\mathbf{L}^1, \mathbf{L}^\infty, \mathbf{L}_0^\infty$ Thin spaces Thick spaces Cutting and splicing at 0 The classes $\mathcal{T}(X), \mathcal{T}^-(X)$	
24.	Spaces of continuous functions	76
	\mathcal{FC} -spaces The class $\mathcal{FC}(X)$ \mathcal{TC}^- -spaces on $J = R_+$ and \mathcal{FC} -spaces on $J = R$	
25.	Notes to Chapter 2	83

Chapter 3. Linear differential equations

30.	Introduction	84
	Summary of the chapter Primitives	
31.	Solutions	86
	Existence, uniqueness, and formulas for the solutions Bounds for the solutions The closedness theorem	
32.	Associate equations in coupled spaces	89
	Associate operator-valued functions Associate equations Green's Formula	
33.	\mathbf{D} -solutions of homogeneous equations	92
	\mathbf{D} -solutions and their initial values Examples and comments A result on associate equations	
34.	Notes to Chapter 3	97

PART II

Chapter 4. Dichotomies

40.	Introduction	101
41.	Ordinary dichotomies	102
	Definition Dichotomies and solutions in \mathcal{T} -spaces Examples	

42.	Exponential dichotomies	110
	Definition Exponential dichotomies and solutions in \mathcal{F} -spaces Example	
43.	Dichotomies for associate equations	117
	Dichotomies for associate equations Ordinary dichotomies and the manifolds X_0, X_0^*	
44.	Finite-dimensional space	120
45.	Notes to Chapter 4	122

Chapter 5. **Admissibility and related concepts**

50.	Introduction	124
	Summary of the chapter Pairs of Banach function spaces	
51.	Admissibility	126
	Definition and boundedness theorem Regular admissibility Admissibility and local closure Some remarks on the admissibility of \mathcal{F} -pairs and related pairs on R_+ Sets of admissible pairs Inadmissible pairs Equations with scalar A on R_+	
52.	(\mathbf{B}, \mathbf{D}) -manifolds	138
	Summary of the chapter (concluded) (\mathbf{B}, \mathbf{D}) -manifolds (\mathbf{B}, \mathbf{D}) -manifolds and admissibility (\mathbf{B}, \mathbf{D}) -subspaces \mathcal{F} -pairs and related pairs Sets of pairs	
53.	(\mathbf{B}, \mathbf{D}) -manifolds, admissibility, and the associate equations	149
	The polar manifold of a (\mathbf{B}, \mathbf{D}) -manifold A result on admissible \mathcal{F} -pairs	
54.	(\mathbf{B}, \mathbf{D}) -subspaces and the associate equations	155
	The polar manifold of a (\mathbf{B}, \mathbf{D}) -subspace Implications of admissibility for the adjoint equation Sets of (\mathbf{B}, \mathbf{D}) -manifolds and -subspaces for \mathcal{F} -pairs and related pairs	
55.	Finite-dimensional space	160
56.	Notes to Chapter 5	162
	Historical notes Complemented $(\mathbf{B}, \mathbf{L}^\infty)$ -subspaces	

Chapter 6. **Admissibility and dichotomies**

60.	Introduction	165
61.	The fundamental inequalities	167

62.	Predichotomy behavior of the solutions of the homogeneous equation	170
	Means and slices of solutions Pointwise nonuniform properties of solutions Miscellaneous corollaries	
63.	Admissibility, (\mathbf{B}, \mathbf{D}) -subspaces, and dichotomies: the general case	179
	Ordinary dichotomies Exponential dichotomies Sets of pairs	
64.	Admissibility, (\mathbf{B}, \mathbf{D}) -subspaces, and dichotomies: the equation with $A \in \mathbf{M}(\tilde{X})$	188
	The main theorems Sets of pairs	
65.	Examples and comments	192
	Examples with constant A Counterexamples for the direct theorems Counterexamples for the converse theorems Examples in infinite-dimensional space Estimation of dichotomy parameters	
66.	Behavior of the solutions of the associate homogeneous equation	211
	Implications of the existence of a (\mathbf{B}, \mathbf{D}) -subspace Implications of the existence of a mere (\mathbf{B}, \mathbf{D}) -manifold A question about dichotomies	
67.	Notes to Chapter 6	221
Chapter 7. Dependence on A		
70.	Introduction	223
71.	Admissibility classes and (\mathbf{B}, \mathbf{D}) -subspaces	224
	Admissibility classes (\mathbf{B}, \mathbf{D}) -subspaces	
72.	Dichotomy classes	237
	Exponential dichotomies Ordinary dichotomies	
73.	Connection in dichotomy classes: Banach spaces	245
	Deformation families Connection by arcs in dichotomy classes	
74.	Connection in dichotomy classes: Hilbert space	251
	A bit of motivation Two geometrical lemmas Deformation families Exponential dichotomies: the general case Exponential dichotomies: the exceptional case Ordinary dichotomies Finite-dimensional space	
75.	Notes to Chapter 7	269

Chapter 8. **Equations on R**

80. Introduction	271
81. (B, D) -dihedra and admissibility	273
The fundamental theorems Some further results	
82. Double dichotomies. Connections with admissibility and (B, D) -dihedra	279
Double dichotomies Examples Connections with admissibility and (B, D) -dihedra Predichotomy behavior of the solutions of the homogeneous equation	
83. Associate equations	293
84. Dependence on A	296
Admissibility classes and closed (B, D) -dihedra Double dichotomy classes Connection in double-dichotomy classes	

PART IIIChapter 9. **Ljapunov's method**

90. Introduction	311
Summary of the chapter Pointwise properties of the solutions. Exceptional sets	
91. Ljapunov functions	316
Ljapunov functions Total derivatives	
92. Exponential dichotomies	320
93. Ordinary dichotomies	327
94. Notes to Chapter 9	332

Chapter 10. **Equations with almost periodic A**

100. Introduction	333
Summary of the chapter Spaces of almost periodic functions Almost periodic equations and solutions. Preliminary facts	
101. The condition $X_{0*\alpha} = \{0\}$	338
102. Exponential dichotomies	341

103. Reflexive and finite-dimensional spaces	343
104. Notes to Chapter 10	345
Equations on R_+ The theory of Favard	
 Chapter 11. Equations with periodic A	
110. Introduction	348
Summary of the chapter Spaces of periodic functions Properties of U	
111. Floquet representation	351
112. Periodic equations and periodic solutions	354
113. The solutions of the homogeneous equation	358
D-solutions Double dichotomies Examples Exponential and double exponential dichotomies	
114. Individual periodic equations	369
 Chapter 12. Higher-order equations	
120. Introduction	373
Summary of the chapter n th primitive functions	
121. The $(m + 1)$ st-order equation	376
122. Admissibility and (\mathbf{B}, \mathbf{D}) -manifolds	381
123. The main theorems	386
REFERENCES	393
INDEX - Author and subject	399
Notation	402