

STUDENT MATHEMATICAL LIBRARY
Volume 89

A First Journey through Logic

Martin Hils
François Loeser

Contents

Introduction	ix
Chapter 1. Counting to Infinity	1
Introduction	1
§1.1. Naive Set Theory	1
§1.2. The Cantor and Cantor-Bernstein Theorems	2
§1.3. Orders	3
§1.4. Operations on Orders	5
§1.5. Ordinal Numbers	7
§1.6. Ordinal Arithmetic	11
§1.7. The Axiom of Choice	14
§1.8. Cardinal Numbers	14
§1.9. Operations on Cardinals	16
§1.10. Cofinality	19
§1.11. Exercises	22
§1.12. Appendix: Hindman's Theorem	28
Chapter 2. First-order Logic	33
Introduction	33

§2.1. Languages and Structures	34
§2.2. Terms and Formulas	36
§2.3. Semantics	39
§2.4. Substitution	41
§2.5. Universally Valid Formulas	45
§2.6. Formal Proofs and Gödel's Completeness Theorem	48
§2.7. Exercises	58
Chapter 3. First Steps in Model Theory	65
Introduction	65
§3.1. Some Fundamental Theorems	66
§3.2. The Diagram Method	70
§3.3. Expansions by Definition	72
§3.4. Quantifier Elimination	75
§3.5. Algebraically Closed Fields	78
§3.6. Ax's Theorem	81
§3.7. Exercises	83
Chapter 4. Recursive Functions	89
Introduction	89
§4.1. Primitive Recursive Functions	90
§4.2. The Ackermann Function	94
§4.3. Partial Recursive Functions	96
§4.4. Turing Computable Functions	98
§4.5. Universal Functions	107
§4.6. Recursively Enumerable Sets	109
§4.7. Elimination of Recursion	113
§4.8. Exercises	115
Chapter 5. Models of Arithmetic and Limitation Theorems	119
Introduction	119
§5.1. Coding Formulas and Proofs	120

§5.2. Decidable Theories	122
§5.3. Peano Arithmetic	125
§5.4. The Theorems of Tarski and Church	131
§5.5. Gödel's First Incompleteness Theorem	133
§5.6. Definability of Satisfiability for Σ_1 -formulas	134
§5.7. Gödel's Second Incompleteness Theorem	137
§5.8. Exercises	141
Chapter 6. Axiomatic Set Theory	147
Introduction	147
§6.1. The Framework	147
§6.2. The Zermelo-Fraenkel Axioms	148
§6.3. The Axiom of Choice	155
§6.4. The von Neumann Hierarchy and the Axiom of Foundation	157
§6.5. Some Results on Incompleteness, Independence and Relative Consistency	161
§6.6. A Glimpse of Further Independence and Relative Consistency Results	169
§6.7. Exercises	173
Bibliography	179
Index	181