Contents

1. Introduction 6
2. Preliminaries and notation 8
   2.1. Notation 8
   2.2. Differential calculus of maps between locally convex spaces 8
   2.3. Fréchet differentiability 9
3. Weighted function spaces 10
   3.1. Definition and examples 10
   3.2. Topological and uniform structure 12
      3.2.1. Reduction to lower order 12
      3.2.2. Projective limits and the topology of $\mathcal{C}^\infty_{\mathcal{W}}(U,Y)$ 13
      3.2.3. A completeness criterion 14
   3.3. Composition on weighted functions and superposition operators 17
      3.3.1. Composition with a multilinear map 17
      3.3.2. Composition of weighted functions with bounded functions 21
      3.3.3. Composition of weighted functions with an analytic map 26
   3.4. Weighted maps into locally convex spaces 30
      3.4.1. Definition and topological structure 30
      3.4.2. Weighted decreasing maps 33
      3.4.3. Composition and superposition 35
4. Lie groups of weighted diffeomorphisms 41
   4.1. Weighted diffeomorphisms and endomorphisms 41
      4.1.1. Composition of weighted endomorphisms in charts 42
      4.1.2. Smooth monoids of weighted endomorphisms 47
   4.2. Lie group structures on weighted diffeomorphisms 48
      4.2.1. The Lie group structure of $\mathrm{Diff}_{\mathcal{W}}(X)$ 48
      4.2.2. On decreasing weighted diffeomorphisms and dense subgroups 54
      4.2.3. On diffeomorphisms that are weighted endomorphisms 55
   4.3. Regularity 57
      4.3.1. The tangent group and the regularity differential equation of $\mathrm{Diff}_{\mathcal{W}}(X)$ 57
      4.3.2. Conclusion and calculation of one-parameter groups 61
5. Integration of certain Lie algebras of vector fields 62
   5.1. On the smoothness of the conjugation action on $\mathrm{Diff}_{\mathcal{W}}(X)_0$ 63
      5.1.1. Contravariant composition on weighted functions 64
   5.2. Conclusion and examples 67
6. Lie group structures on weighted mapping groups 69
   6.1. Weighted maps into Banach Lie groups 70
      6.1.1. Construction of the Lie group 70
      6.1.2. Regularity 74
      6.1.3. Semidirect products with weighted diffeomorphisms 77

6.2. Weighted maps into locally convex Lie groups 78
6.2.1. Construction of the Lie group 78
6.2.2. A larger Lie group of weighted mappings 81
A. Differential calculus 95
A.1. Differential calculus of maps between locally convex spaces 95
A.1.1. Curves and integrals 95
A.1.2. Differentiable maps 98
A.2. Fréchet differentiability 104
A.3. Relation between the differential calculi 108
A.4. Some facts concerning ordinary differential equations 111
A.4.1. Maximal solutions of ODEs 111
A.4.2. Flows and dependence on parameters and initial values 113
B. Locally convex Lie groups 116
B.1. Locally convex manifolds 116
B.2. Lie groups 117
B.2.1. Generation of Lie groups 118
B.2.2. Regularity 118
B.2.3. Group actions 119
C. Quasi-inversion in algebras 120
C.1. Definition 121
C.2. Topological monoids and algebras with continuous quasi-inversion 121
References 124
Notation 126
Index 128