

# Contents

Chapter 1. Introduction	1
1.1. Description of the Results	1
1.2. Notations, Terminology and Conventions	5
Acknowledgments	7
Chapter 2. Stabilizations of $\mathbb{E}_\infty$ Operads	9
2.1. Kan Suspensions and Moore Loopings	9
2.2. Operadic (De)suspensions	13
2.3. The Stable Eilenberg-Zilber Operad	15
2.4. The Stable McClure-Smith Operad	18
2.5. The Stable Barratt-Eccles Operad	21
2.6. The (Co)homologies, in Individual Arities, of the Stable Operads	24
2.7. Comparison with the Commutativity and Constant Operads	25
Chapter 3. Homotopy Theory of Algebras Over the Stable Operads	29
3.1. Cell Algebras	29
3.2. Enveloping Operads	30
3.3. Admissibility and Semi-admissibility of Operads	33
3.4. The Homotopy Theory of Algebras over the Stable Operads	35
Chapter 4. Cohomology Operations for Algebras Over the Stable Operads	41
4.1. The Cohomology Operations I	41
4.2. The Completion $\widehat{\mathcal{B}}$ of the Algebra of Generalized Steenrod Operations	46
4.3. Cohomologies of Free Algebras over the Stable Operads	55
4.4. The Cohomology Operations II	62
4.5. Unstable Modules over $\widehat{\mathcal{B}}$	63
4.6. The Relation Between $\widehat{\mathcal{B}}$ and the Steenrod Algebra $\mathcal{A}$	64
4.7. Comparison of the Homotopy Theories of Algebras Over $\mathcal{E}_{\text{st}}^\dagger$ and $\mathcal{M}_{\text{st}}^\dagger$	66
Chapter 5. The Homotopy Coherent, or $\infty$ -, Additivity of the Stable Operads	69
5.1. Derived Coproducts of Algebras Over the Stable Operads	69
5.2. Derived Pushouts of Algebras Over the Stable Operads	73
Chapter 6. A Spectral Cochains Adjunction	77
6.1. Spectra and Their Model Structure	77
6.2. Spectral Cochains as Algebras Over the Stable Operads	80
6.3. Change of Coefficients from $\mathbb{F}_p$ to $\overline{\mathbb{F}}_p$	85
6.4. The Adjoint to Spectral Cochains	90
Chapter 7. Algebraic Models of $p$ -Adic Stable Homotopy Types	99
7.1. The Case of the Generalized Eilenberg-MacLane Spectra $\Sigma^n \mathbf{HF}_p$	99

7.2. Fibration Theorems	104
7.3. Proof of the Main Theorem	107
Bibliography	109
Index	111