

Contents

Preface	xi
1 Random Variables	1
1.1 Motivation	1
1.2 Discrete Random Variables	1
1.3 Continuous Random Variables	3
1.4 Expectation and Variance	5
1.5 Markov and Lyapunov Inequalities	8
1.6 Independence	9
1.7 Normal Random Variables	9
1.8 Central Limit Theorem	11
1.9 Strong Law of Large Numbers	11
1.10 Notes and References	12
1.11 Program of Chapter 1 and Walk-through	14
2 Computer Simulations	17
2.1 Motivation	17
2.2 Pseudorandom Number Generation	17
2.3 Monte Carlo Method	18
2.4 Kernel Density Estimation	21
2.5 Notes and References	23
2.6 Program of Chapter 2 and Walk-through	25
3 Brownian Motion	27
3.1 Motivation	27
3.2 Defining Brownian Motion	27
3.3 Discretized Brownian Motion	28
3.4 Filling in a Brownian path	29
3.5 A Scaling Property	31
3.6 Nondifferentiability and Unboundedness of Variation	32
3.7 Fourier Series Representation	34
3.8 White Noise Interpretation	35
3.9 Notes and References	37
3.10 Program of Chapter 3 and Walk-through	38
4 Stochastic Integrals	41
4.1 Motivation	41
4.2 Itô and Stratonovich Integrals	41
4.3 Properties of the Itô Integral	44

4.4	“ $dW^2 = dt$ ”	46
4.5	Notes and References	48
4.6	Program of Chapter 4 and Walk-through	49
5	Stochastic Differential Equations	51
5.1	Motivation	51
5.2	Stochastic Differential Equations	51
5.3	Examples of Stochastic Differential Equations	53
5.4	Existence of Solutions	58
5.5	Weak versus Strong Solutions	58
5.6	Notes and References	59
5.7	Program of Chapter 5 and Walk-through	60
6	The Itô Formula	63
6.1	Motivation	63
6.2	The Itô Formula	63
6.3	Using the Itô Formula	65
6.4	Notes and References	67
6.5	Program of Chapter 6 and Walk-through	68
7	Itô versus Stratonovich	71
7.1	Motivation	71
7.2	The Stratonovich SDE	71
7.3	Which Is Better?	74
7.4	Notes and References	74
7.5	Program of Chapter 7 and Walk-through	76
8	Euler–Maruyama	79
8.1	Motivation	79
8.2	Euler–Maruyama	79
8.3	Weak Convergence	80
8.4	Strong Convergence	83
8.5	Notes and References	85
8.6	Program of Chapter 8 and Walk-through	87
9	Weak Convergence	91
9.1	Motivation	91
9.2	Feynman–Kac Formula	91
9.3	Outline of Weak Proof	93
9.4	Notes and References	96
9.5	Program of Chapter 9 and Walk-through	98
10	Strong Convergence	101
10.1	Motivation	101
10.2	Setting Up the Proof	101
10.3	Outline of Strong Proof	102
10.4	Remarks about the Proof	104
10.5	Nonconvergence of EM	104
10.6	Notes and References	106
10.7	Program of Chapter 10 and Walk-through	108

11	Implicit Methods and Numerical Stability	111
11.1	Motivation	111
11.2	Stochastic θ -Method	112
11.3	Linear Test Equation	112
11.4	Mean-Square Stability of the θ -Method	113
11.5	Asymptotic Stability of the θ -Method	115
11.6	Notes and References	118
11.7	Program of Chapter 11 and Walk-through	121
12	Mean Exit Times	123
12.1	Motivation	123
12.2	Background	123
12.3	Monte Carlo for Mean Exit Time	123
12.4	Computational Experiments	124
12.5	Notes and References	130
12.6	Program of Chapter 12 and Walk-through	134
13	Exotic Options	137
13.1	Motivation	137
13.2	Introduction	137
13.3	Path-Dependent Options	139
13.4	Asset Price Models	144
13.5	Monte Carlo for Expected Payoff	144
13.6	Notes and References	146
13.7	Program of Chapter 13 and Walk-through	148
14	Steady States	151
14.1	Motivation	151
14.2	Meet the Fokker–Planck Equation	151
14.3	Computing to a Steady State	153
14.4	Computations with a Population Model	156
14.5	A Nonsmooth Example	161
14.6	Notes and References	164
14.7	Program of Chapter 14 and Walk-through	168
15	Multilevel Monte Carlo	171
15.1	Motivation	171
15.2	Complexity of Monte Carlo for SDEs	171
15.3	Multilevel Monte Carlo	173
15.4	Notes and References	176
15.5	Program of Chapter 15 and Walk-through	179
16	Jumps	181
16.1	Motivation	181
16.2	Poisson Process	181
16.3	A Jump-SDE	185
16.4	More General Jump-SDEs	187
16.5	Linear Stability	189
16.6	Notes and References	191
16.7	Program of Chapter 16 and Walk-through	192

17 Higher-Order Methods	195
17.1 Motivation	195
17.2 The EM Method	196
17.3 The Stochastic Heun Method	196
17.4 The Milstein Method	198
17.5 The Milstein Method with Several Brownian Motions	200
17.6 Higher-Order Taylor Methods	202
17.7 Talay–Tubaro Extrapolation for SDEs	202
17.8 Notes and References	203
17.9 Program of Chapter 17 and Walk-through	204
18 Systems of Stochastic Differential Equations	207
18.1 Motivation	207
18.2 Systems of SDEs	207
18.3 Examples of SDE systems	209
18.4 Itô Formula for a System of SDEs	217
18.5 Notes and References	218
18.6 Program of Chapter 18 and Walk-through	220
19 Numerical Methods For Systems	223
19.1 Motivation	223
19.2 EM for SDE Systems	223
19.3 Milstein for SDE Systems	224
19.4 Taylor Expansions for SDE systems	228
19.5 Notes and References	231
19.6 Program of Chapter 19 and Walk-through	233
20 Chemical Kinetics	235
20.1 Motivation	235
20.2 Stoichiometrics and Propensities	235
20.3 Chemical Master Equation	238
20.4 Stochastic Simulation Algorithm	239
20.5 Tau-Leaping	242
20.6 Chemical Langevin Equation	244
20.7 Reaction Rate Equation	246
20.8 Notes and References	249
20.9 Program of Chapter 20 and Walk-through	253
List of Symbols	257
Bibliography	259
Index	273