

A SYNOPSIS
OF
ELEMENTARY RESULTS
IN
PURE AND APPLIED MATHEMATICS :

CONTAINING
PROPOSITIONS, FORMULÆ, AND METHODS OF ANALYSIS,
WITH
ABRIDGED DEMONSTRATIONS.

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TABLE OF CONTENTS.



PART II.

	No. of Article.
SECTION VIII.—DIFFERENTIAL CALCULUS.	
INTRODUCTION	1400
Successive differentiation	1405
Infinitesimals. Differentials	1407
DIFFERENTIATION.	
Methods... ..	1411-21
SUCCESSIVE DIFFERENTIATION—	
Leibnitz's theorem	1460
Derivatives of the n th order (see Index)	1461-71
PARTIAL DIFFERENTIATION	1480
THEORY OF OPERATIONS	1483
Distributive, Commutative, and Index laws... ..	1488
EXPANSION OF EXPLICIT FUNCTIONS—	
Taylor's and Maclaurin's theorems	1500, 1507
Symbolic forms of the same	1520-3
$f(x+h, y+k)$, &c.	1512-4
Methods of expansion by indeterminate coefficients.	
Four rules... ..	1527-34
Method by Maclaurin's theorem	1524
Arbogast's method of expanding $\phi(z)$	1536
Bernoulli's numbers	1539
Expansions of $\phi(x+h) - \phi(x)$. Stirling and Boole	1546-7
EXPANSIONS OF IMPLICIT FUNCTIONS—	
Lagrange's, Laplace's, and Burmann's theorems, 1552, 1556-63	
Cayley's series for $\frac{1}{\phi(z)}$	1555
Abel's series for $\phi(x+a)$	1572
INDETERMINATE FORMS	1580
JACOBIANS	1600
Modulus of transformation	1604

	No. of Article.
QUANTICS	1620
Euler's theorem	1621
Eliminant, Discriminant, Invariant, Covariant, Hessian	1626-30
Theorems concerning discriminants	1635-45
Notation $A \equiv bc - f^2$, &c.	1642
Invariants	1648-52
Cogredients and Emanents	1653-5
IMPLICIT FUNCTIONS—	
One independent variable	1700
Two independent variables	1725
n independent variables	1737
CHANGE OF THE INDEPENDENT VARIABLE	1760
Linear transformation	1794
Orthogonal transformation	1799
Contragredient and Contravariant	1813
Notation $z_x \equiv p$, &c., q, r, s, t	1815
MAXIMA AND MINIMA—	
One independent variable	1830
Two independent variables	1841
Three or more independent variables... ..	1852
Discriminating cubic	1849
Method of undetermined multipliers	1862
Continuous maxima and minima	1866

SECTION IX.—INTEGRAL CALCULUS.

INTRODUCTION	1900
Multiple Integrals	1905
METHODS OF INTEGRATION—	
By Substitution, Parts, Division, Rationalization, Partial fractions, Infinite series	1908-19
STANDARD INTEGRALS	1921
VARIOUS INDEFINITE INTEGRALS—	
Circular functions	1954
Exponential and logarithmic functions	1998
Algebraic functions	2007
Integration by rationalization... ..	2110
Integrals reducible to Elliptic integrals	2121-47
Elliptic integrals approximated to	2127
SUCCESSIVE INTEGRATION	2148
HYPERBOLIC FUNCTIONS $\cosh x, \sinh x, \tanh x$	
Inverse relations	2210
Geometrical meaning of $\tanh S$	2213
Logarithm of an imaginary quantity	2214

	No. of Article.
DEFINITE INTEGRALS—	
Summation of series	2230
THEOREMS RESPECTING LIMITS OF INTEGRATION	2233
METHODS OF EVALUATING DEFINITE INTEGRALS (<i>Eight rules</i>) ...	2245
DIFFERENTIATION UNDER THE SIGN OF INTEGRATION	2253
Integration by this method	2258
Change in the order of integration	2261
APPROXIMATE INTEGRATION BY BERNOULLI'S SERIES	2262
THE INTEGRALS $B(l, m)$ and $\Gamma(n)$	2280
$\log \Gamma(1+n)$ in a converging series	2294
Numerical calculation of $\Gamma(x)$	2317
INTEGRATION OF ALGEBRAIC FORMS	2341
INTEGRATION OF LOGARITHMIC AND EXPONENTIAL FORMS ...	2391
INTEGRATION OF CIRCULAR FORMS	2451
INTEGRATION OF CIRCULAR LOGARITHMIC AND EXPONENTIAL FORMS	2571
MISCELLANEOUS THEOREMS—	
Frullani's, Poisson's, Abel's, Kummer's, and Cauchy's formulæ	2700–13
FINITE VARIATION OF A PARAMETER	2714
Fourier's formula	2726
THE FUNCTION $\psi(x)$	2743
Summation of series by the function $\psi(x)$	2757
$\psi(x)$ as a definite integral independent of $\psi(1)$...	2766
NUMERICAL CALCULATION OF $\log \Gamma(x)$	2771
CHANGE OF THE VARIABLES IN A DEFINITE MULTIPLE INTEGRAL	2774
MULTIPLE INTEGRALS—	
EXPANSIONS OF FUNCTIONS IN CONVERGING SERIES—	
Derivatives of the n th order	2852
Miscellaneous expansions	2911
Legendre's function X_n	2936
EXPANSION OF FUNCTIONS IN TRIGONOMETRICAL SERIES ...	2955
APPROXIMATE INTEGRATION	2991
Methods by Simpson, Cotes, and Gauss	2992–7

SECTION X.—CALCULUS OF VARIATIONS.

FUNCTIONS OF ONE INDEPENDENT VARIABLE	3028
Particular cases... ..	3033
Other exceptional cases	3045
FUNCTIONS OF TWO DEPENDENT VARIABLES	3051
Relative maxima and minima	3069
Geometrical applications	3070

	No. of Article.
FUNCTIONS OF TWO INDEPENDENT VARIABLES	3075
Geometrical applications	3078
APPENDIX—	
On the general object of the Calculus of Variations...	3084
Successive variation	3087
Immediate integrability	3090

SECTION XI.—DIFFERENTIAL EQUATIONS.

GENERATION OF DIFFERENTIAL EQUATIONS	3150
DEFINITIONS AND RULES... ..	3158
SINGULAR SOLUTIONS	3168
FIRST ORDER LINEAR EQUATIONS	3184
Integrating factor for $Mdx + Ndy = 0$	3192
Riccati's Equation	3214
FIRST ORDER NON-LINEAR EQUATIONS... ..	3221
Solution by factors	3222
Solution by differentiation	3236
HIGHER ORDER LINEAR EQUATIONS	3237
Linear Equations with Constant Coefficients	3238
HIGHER ORDER NON-LINEAR EQUATIONS	3251
Depression of Order by Unity... ..	3262
EXACT DIFFERENTIAL EQUATIONS	3270
MISCELLANEOUS METHODS	3276
Approximate solution of Differential Equations by Taylor's theorem	3289
SINGULAR SOLUTIONS OF HIGHER ORDER EQUATIONS	3301
EQUATIONS WITH MORE THAN TWO VARIABLES... ..	3320
SIMULTANEOUS EQUATIONS WITH ONE INDEPENDENT VARIABLE... ..	3340
PARTIAL DIFFERENTIAL EQUATIONS	3380
Linear first order P. D. Equations	3381
Non-linear first order P. D. Equations	3399
Non-linear first order P. D. Equations with more than two independent variables	3409
SECOND ORDER P. D. EQUATIONS	3420
LAW OF RECIPROCITY	3446
SYMBOLIC METHODS	3470
SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS BY SERIES	3604
SOLUTION BY DEFINITE INTEGRALS	3617
P. D. EQUATIONS WITH MORE THAN TWO INDEPENDENT VARIABLES	3629
DIFFERENTIAL RESOLVENTS OF ALGEBRAIC EQUATIONS	3631

SECTION XII. — CALCULUS OF FINITE
DIFFERENCES.

FORMULÆ FOR FIRST AND n th DIFFERENCES	3706
Expansion by factorials	3730
Generating functions	3732
The operations E , Δ , and d_x	3735
Herschel's theorem	3757
A theorem conjugate to Maclaurin's	3759
INTERPOLATION	3762
Lagrange's interpolation formula	3768
MECHANICAL QUADRATURE	3772
Cotes's and Gauss's formulæ	3777
Laplace's formula	3778
SUMMATION OF SERIES	3781
APPROXIMATE SUMMATION	3820

SECTION XIII.—PLANE COORDINATE GEOMETRY.

SYSTEMS OF COORDINATES—

Cartesian, Polar, Trilinear, Areal, Tangential, and Intercept Coordinates	4001-28
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ANALYTICAL CONICS IN CARTESIAN COORDINATES.

LENGTHS AND AREAS	4032
TRANSFORMATION OF COORDINATES	4048
THE RIGHT LINE	4052
Equations of two or more right lines... ..	4110
GENERAL METHODS	4114
Poles and Polars	4124
THE CIRCLE	4136
Co-axal circles	4161
THE PARABOLA	4200
THE ELLIPSE AND HYPERBOLA	4250
Right line and ellipse	4310
Polar equations of the conic	4336
Conjugate diameters	4346
Determination of various angles	4375
THE HYPERBOLA REFERRED TO ITS ASYMPTOTES	4387
The rectangular hyperbola	4392

	No. of Article.
THE GENERAL EQUATION	4400
The ellipse and hyperbola	4402
Invariants of the conic	4417
The parabola	4430
Method without transformation of the axes	4445
Rules for the analysis of the general equation	4464
Right line and conic with the general equation	4487
Intercept equation of a conic	4498
SIMILAR CONICS	4522
CIRCLE OF CURVATURE—	
Contact of Conics	4527
CONFOCAL CONICS	4550
ANALYTICAL CONICS IN TRILINEAR COORDINATES.	
THE RIGHT LINE	4601
Equations of particular lines and coordinate ratios of particular points in the trigon	4628
ANHARMONIC RATIO	4648
The complete quadrilateral	4652
THE GENERAL EQUATION OF A CONIC	4656
Director-Circle	4693
PARTICULAR CONICS	4697
Conic circumscribing the trigon	4724
Inscribed conic of the trigon	4739
Inscribed circle of the trigon	4747
General equation of the circle... ..	4751
Nine-point circle	4754
Triplicate-ratio circle	4754 <i>b</i>
Seven-point circle	4754 <i>c</i>
CONIC AND SELF-CONJUGATE TRIANGLE... ..	4755
On lines passing through imaginary points	4761
CARNOT'S, PASCAL'S, AND BRIANCHON'S THEOREMS	4778-83
THE CONIC REFERRED TO TWO TANGENTS AND THE CHORD OF CONTACT—	
Related conics	4803
ANHARMONIC PENCILS OF CONICS	4809
CONSTRUCTION OF CONICS	4822
Newton's method of generating a conic	4829
Maclaurin's method of generating a conic	4830
THE METHOD OF RECIPROCAL POLARS... ..	4844
TANGENTIAL COORDINATES	4870
Abridged notation	4907

	No. of Article.
ON THE INTERSECTION OF TWO CONICS—	
Geometrical meaning of $\sqrt{-1}$	4916
THE METHOD OF PROJECTION	4921
INVARIANTS AND COVARIANTS	4936
To find the foci of the general conic	5008

THEORY OF PLANE CURVES.

TANGENT AND NORMAL	5100
RADIUS OF CURVATURE AND EVOLUTE	5134
INVERSE PROBLEM AND INTRINSIC EQUATION	5160
ASYMPTOTES	5167
Asymptotic curves	5172
SINGULARITIES OF CURVES—	
Concavity and Convexity	5174
Points of inflexion, multiple points, &c.	5175-87
CONTACT OF CURVES	5188
ENVELOPES	5192
INTEGRALS OF CURVES AND AREAS	5196
INVERSE CURVES... ..	5212
PEDAL CURVES	5220
ROULETTES	5229
Area, length, and radius of curvature... ..	5230-5
The envelope of a carried curve	5239
Instantaneous centre	5243
Holditch's theorem	5244
Trajectories	5246
Curves of pursuit	5247
Caustics... ..	5248
Quetelet's theorem	5249
TRANSCENDENTAL AND OTHER CURVES—	
The cycloid	5250
The companion to the cycloid... ..	5258
Prolate and curtate cycloids	5260
Epitrochoids and hypotrochoids	5262
Epicycloids and hypocycloids... ..	5266
The Catenary	5273
The Tractrix	5279
The Syntractrix	5282
The Logarithmic Curve	5284
The Equiangular Spiral	5288
The Spiral of Archimedes	5296
The Hyperbolic or Reciprocal Spiral... ..	5302

	No. of Article.
The Involute of the Circle	5306
The Cissoid	5309
The Cassinian or Oval of Cassini	5313
The Lemniscate	5317
The Conchoid	5320
The Limaçon	5327
The Versiera (or Witch of Agnesi)	5335
The Quadratrix... ..	5338
The Cartesian Oval	5341
The semi-cubical parabola	5359
The folium of Descartes	5360
LINKAGES AND LINKWORK	5400
Kempe's five-bar linkage. Eight cases5401-5417
Reversor, Multiplier, and Translator	5407
Peaucellier's linkage	5410
The six-bar inverter	5419
The eight-bar double inverter	5420
The Quadruplane or Versor Inverter	5422
The Pentograph or Proportionator	5423
The Isoklinostat or Angle-divider	5425
A linkage for drawing an Ellipse	5426
A linkage for drawing a Limaçon, and also a bicir- cular quartic	5427
A linkage for solving a cubic equation	5429
On three-bar motion in a plane	5430
The Mechanical Integrator	5450
The Planimeter	5452

SECTION XIV —SOLID COORDINATE GEOMETRY.

SYSTEMS OF COORDINATES	5501
THE RIGHT LINE	5507
THE PLANE	5545
TRANSFORMATION OF COORDINATES	5574
THE SPHERE	5582
The Radical Plane	5585
Poles of similitude	5587
CYLINDRICAL AND CONICAL SURFACES	5590
Circular Sections	5596
ELLIPSOID, HYPERBOLOID, AND PARABOLOID5599-5621

	No. of Article.
CENTRAL QUADRIC SURFACE—	
Tangent and diametral planes... ..	5626
Eccentric values of the coordinates	5638
CONFOCAL QUADRICS	5656
Reciprocal and Enveloping Cones	5664
THE GENERAL EQUATION OF A QUADRIC	5673
RECIPROCAL POLARS	5704
THEORY OF TORTUOUS CURVES... ..	5721
The Helix	5756
GENERAL THEORY OF SURFACES—	
General equation of a surface	5780
Tangent line and cone at a singular point	5783
The Indicatrix Conic	5795
Euler's and Meunier's theorems	5806-9
Curvature of a surface... ..	5826
Osculating plane of a line of curvature	5835
GEODESICS	5837-48
INVARIANTS	5856
INTEGRALS FOR VOLUMES AND SURFACES	5871
Guldin's rules	5879
CENTRE OF MASS	5884
MOMENTS AND PRODUCTS OF INERTIA	5903
Momental ellipsoids	5925-40
Momental ellipse	5953
Integrals for moments of inertia	5978
PERIMETERS, AREAS, VOLUMES, CENTRES OF MASS, AND MOMENTS OF INERTIA OF VARIOUS FIGURES—	
Rectangular lamina and Right Solid... ..	6015
The Circle	6019
The Right Cone	6043
Frustum of Cylinder	6048
The Sphere	6050
The Parabola	6067
The Ellipse	6083
Fagnani's, Griffith's, and Lambert's theorems	6088-6114
The Hyperbola	6115
The Paraboloid	6126
The Ellipsoid	6142
Prolate and Oblate Spheroids... ..	6152-65