Contents

Preface xiii

Introduction 1

Aims 1
Highlights 2
Categories of Usage 3
Structure of the Chapters 4
The Specifications List 5
Structure of the "Next" Programs 6
Structure of the "Random" Programs 7
Arrays and Specifications 8

1 Next Subset of an *n*-Set (NEXSUB) 11

Algorithm NEXSUB 13
Algorithm NS 14
Flow Chart NEXSUB 15
Subroutine Specifications 15
Sample Output 17

2 Random Subset of an *n*-Set (RANSUB) 18

Algorithm RANSUB 18
Flow Chart RANSUB 18

Subroutine Specifications 19
Sample Output 19

3 Next *k*-Subset of an *n*-Set (NEXKSB/NXKSRD) 21

Algorithm NEXKSB (Lexicographic) 22
Flow Chart NXKSRD 26
Flow Chart NEXKSB (Lexicographic Order) 27
Subroutine Specifications 28
Sample Output 29
Subroutine Specifications 30
Sample Output 32

4 Random *k*-Subset of an *n*-Set (RANKSB) 35

Algorithm RANKSB 37
Algorithm RKS2 37
Subroutine Specifications 38
Sample Output 39

5 Next Composition of *n* into *k* Parts (NEXCOM) 40

Algorithm NEXCOM 43
Flow Chart NEXCOM 44
Subroutine Specifications 45
Sample Output 46

6 Random Composition of *n* into *k* Parts (RANCOM) 47

Algorithm RANCOM 47
Subroutine Specifications 48

7 Next Permutation of *n* Letters (NEXPER) 49

Algorithm NEXPER 55
Flow Chart NEXPER 56
Subroutine Specifications 57
Sample Output 58

8 Random Permutation of *n* Letters (RANPER) 60

Algorithm RANPER 60
Flow Chart RANPER 61
Subroutine Specifications 61
Sample Output 62

9 Next Partition of Integer *n* (NEXPAR) 63

Algorithm NEXPAR 66
Subroutine Specifications 66
Sample Output 68

10 Random Partition of Integer *n* (RANPAR) 70

Algorithm RANPAR 72
Flow Chart RANPAR 73
Subroutine Specifications 74
Sample Output 76

Postscript: Deus ex Machina

11 Next Partition of an *n*-Set (NEXEQU) 81

Algorithm NEXEQU 83
Flow Chart NEXEQU 84
Subroutine Specifications 85
Sample Output 86

12 Random Partition of an *n*-Set (RANEQU) 87

Algorithm RANEQU 88
Flow Chart RANEQU 89
Subroutine Specifications 90
Sample Output 92

13 Renumbering Rows and Columns of an Array (RENUMB) 93

Algorithm TAG 98
Algorithm RENUMB 98
Subroutine Specifications 99
Sample Output 100

14 Spanning Forest of a Graph (SPANFO) 102

Algorithm SPANFO 105
Subroutine Specifications 106
Sample Output 108

15 Newton Forms of a Polynomial (POLY) 110

Algorithm Value 110
Algorithm Newton 111
Algorithm Taylor 112
Algorithm Reverse Stirling 113
Flow Chart POLY 115
Subroutine Specifications 115
Sample Output 117

16 Chromatic Polynomial of a Graph (CHROMP) 118

Flow Chart CHROMP 125
Subroutine Specifications 129
Sample Output 133

17 Composition of Power Series (POWSER) 134

Algorithm POWSER 136
Flow Chart POWSER 138
Subroutine Specifications 139
First Sample Output, Option 1 141
Second Sample Output, Option 1 141
Sample Output, Option 3 142
Sample Output, Option 4 142

18 Network Flows (NETFLO) 143

Subroutine Specifications 148
Sample Output 151

19 The Permanent Function (PERMAN) 152

Computation of the Permanent Function 155
Algorithm PERMAN 159
Subroutine Specifications 159
Sample Output 160

20 Invert a Triangular Array (INVERT) 161

Algorithm INVERT 161
Subroutine Specifications 162

21 Triangular Numbering in Partially Ordered Sets (TRIANG) 163

Algorithm TRIANG 165
Subroutine Specifications 165
Sample Output 166

22 The Möbius Function (MOBIUS) 168

Subroutine Specifications 172
Sample Output 173

23 The Backtrack Method (BACKTR) 175

A. General (BACKTR) 175
Flow Chart BACKTR 179
Subroutine Specifications 180
B. Coloring the Vertices of a Graph (COLVRT) 181
Subroutine Specifications 182
Sample Output 183
C. Euler Circuits (EULCRC) 184
Algorithm EULCRC 185
Subroutine Specifications 185
Sample Output 187
D. Hamilton Circuits (HAMCRC) 191
Subroutine Specifications 192

Sample Output 1 193
Sample Output 2 195
E. Spanning Trees (SPNTRE) 197
Subroutine Specifications 198
Sample Output 199

24 Random Tree (RANTRE) 202

Flow Chart RANTRE 206
Subroutine Specifications 207
Sample Output 208

25 Random Unlabeled Rooted Trees (RANRUT) 211

Algorithm RANRUT 215
Flow Chart RANRUT 216
Subroutine Specifications 217
Sample Output 219

26 The Sign of a Permutation (SIGNUM) 221

Algorithm SIGNUM 222
Subroutine Specifications 222
Sample Output 223

27 Sorting (HPSORT) 225

Algorithm $\mathscr{F}$ (1, n) 228
Algorithm Toheap 228
Algorithm Sortheap 229
Subroutine Specifications 229
Sample Output 230

28 Tree of Minimal Length (MINSPT) 232

Algorithm MINSPT 234
Flow Chart MINSPT 235
Subroutine Specifications 236
Sample Output 237

Exercises 238

Bibliographic Notes 244

References 247

Index 251