

Dipl.-Ing. Hansjörg Gerhard Sage, Illkirch

# **Synthesis of $H_{\infty}$ Controllers with Application to Industrial Robot Manipulators**

Reihe **8**: Meß-, Steuerungs-  
und Regelungstechnik

Nr. **678**

# CONTENTS

<b>1. Introduction</b>	1
1.1 Linear $\mathcal{H}_\infty$ Theory	2
1.2 Synthesis of Robust Linear $\mathcal{H}_\infty$ Controllers	5
1.2.1 Computation of Linear Robust $\mathcal{H}_\infty$ Controllers	5
1.2.2 Including Performance Criteria into $\mathcal{H}_\infty$ Controller Synthesis	7
1.2.3 Contributions of this Thesis	10
1.3 Robust Control of Robot Manipulators	12
1.3.1 Overview over Existing Approaches	12
1.3.2 Limitations of Existing Approaches	13
1.3.3 Contributions of this Thesis	15
1.4 Organization of this Thesis	15
<b>2. Background Material</b>	16
2.1 Linear Algebra and Linear System Theory	16
2.1.1 Linear Algebra	16
2.1.2 Transfer Functions and State Space Realizations	17
2.1.3 Controllability and Related Concepts	19
2.1.4 Interconnection of Systems	20
2.1.5 Norms and Spaces	22
2.2 General Results on Robust $\mathcal{H}_\infty$ Stabilization	24
2.3 Robust Stabilization of Normalized Left Coprime Factorizations	28
<b>3. Reduced Order <math>\mathcal{H}_\infty</math> Controllers</b>	35
3.1 Preliminaries	35
3.1.1 Linear Algebra and Linear System Theory	36
3.1.2 Algebraic Riccati Equations	38
3.2 $\mathcal{H}_\infty$ Norm Bounded Realizations	39
3.3 Reduced Order $\mathcal{H}_\infty$ Controllers	48
3.3.1 Non-Minimal Elements in a Lower LFT	49
3.3.2 Application to the Computation of Reduced Order $\mathcal{H}_\infty$ Controllers	53
<b>4. Synthesis of Performance Optimized <math>\mathcal{H}_\infty</math> Controllers</b>	55
4.1 Loop Shaping	56
4.1.1 Principle	56
4.1.2 Advantages	59
4.2 Performance Optimized Loop Shaping	60
4.2.1 Design Specifications	60
4.2.2 Definition of the Optimization Problem	62
4.3 Optimization Algorithm	64

4.3.1	Preliminaries . . . . .	64
4.3.2	Sequential Quadratic Programming . . . . .	66
4.3.3	Quadratic Programming . . . . .	67
4.4	Practicalities . . . . .	69
4.4.1	Order Selection of the Filters . . . . .	69
4.4.2	Discrete Time Controllers . . . . .	69
4.4.3	Approximation of the Derivatives . . . . .	70
4.4.4	Design Parameters . . . . .	70
<b>5.</b>	<b>Synthesis of Robust <math>\mathcal{H}_\infty</math> Controllers</b>	
	<b>for Industrial Robot Manipulators . . . . .</b>	<b>71</b>
5.1	Dynamic Model of an Industrial Robot Manipulator . . . . .	72
5.1.1	Actuator Dynamics . . . . .	72
5.1.2	Rigid Robot Dynamics . . . . .	73
5.1.3	Flexible Joints . . . . .	74
5.1.4	Friction Model . . . . .	74
5.1.5	Comprehensive Robot Model . . . . .	76
5.2	The SCEMI 6P01 . . . . .	78
5.3	Controller Synthesis . . . . .	79
5.3.1	Design Specifications . . . . .	79
5.3.2	Nominal Model . . . . .	80
5.3.3	Application of the Performance Optimization . . . . .	81
5.3.4	Optimization Criterion . . . . .	83
5.3.5	Controllers . . . . .	83
5.4	Simulations and Experiments . . . . .	83
<b>6.</b>	<b>Conclusion . . . . .</b>	<b>93</b>
6.1	Main Contributions . . . . .	93
6.2	Directions of Future Work . . . . .	94
	<b>Appendix . . . . .</b>	<b>94</b>
<b>A.</b>	<b>Proofs . . . . .</b>	<b>95</b>
A.1	Proof of Theorem 2.23 . . . . .	95
A.2	Proof of Lemma 3.6 . . . . .	100
A.3	Proof of Lemma 3.7 . . . . .	101
A.4	Proof of Lemma 3.8 . . . . .	104
A.5	Proof of Theorem 3.10 . . . . .	106
A.6	Proof of Theorem 3.12 . . . . .	113
A.7	Proof of Lemma 3.17 . . . . .	122
A.8	Proof of Lemma 3.18 . . . . .	124
<b>B.</b>	<b>The Optimization Algorithm . . . . .</b>	<b>126</b>
<b>C.</b>	<b>Dynamic Model of the SCEMI 6P01 . . . . .</b>	<b>128</b>
C.1	The Denavit-Hartenberg Parameters . . . . .	128
C.2	The Kinetic Energy . . . . .	129
C.3	The Potential Energy . . . . .	131
C.4	The Equation of Motion . . . . .	131

---

C.4.1	The Inertia Matrix . . . . .	131
C.4.2	The Centrifugal and Coriolis torques . . . . .	132
C.4.3	The Gravitational Torques . . . . .	132
C.5	Numerical Values . . . . .	133
<b>D.</b>	<b>Optimization Criteria &amp; Controllers . . . . .</b>	<b>136</b>
D.1	Optimization Criteria . . . . .	136
D.2	Controllers . . . . .	138