

Dipl.-Ing. Christina Benthack, Wachenheim

**Feedback-based
Optimization of a Class
of Constrained Nonlinear
Systems
Application to a Biofilter**

Reihe **8**: Meß-, Steuerungs-
und Regelungstechnik

Nr. **677**

Contents

Nomenclature	VIII
Summary	XI
Version Abrégée	XIII
Kurzfassung	XV
1 Introduction	1
1.1 Motivation	1
1.2 State of the Art	2
1.3 Objectives of the Thesis	5
1.4 Organization of the Thesis	6
2 Preliminaries	7
2.1 Nonlinear Systems	7
2.1.1 Notations	8
2.1.2 Controllability and Accessibility	10
2.1.3 Feedback Linearization	13
2.1.4 Input-Output Linearization	15
2.2 Optimal Control Theory	19
2.2.1 Problem Formulation	20
2.2.2 Pontryagin's Maximum Principle	21
2.2.3 Path Constraints	24
2.2.4 Terminal Constraints	27
2.2.5 Free Terminal Time Problems	27
2.3 Numerical Methods for Optimal Control Problems	28
2.3.1 Numerical Problem Formulation	29
2.3.2 Parameterization of Control and/or State Variables	30
2.3.3 Discussion	32

2.4	Conclusions	34
3	Characterization of Optimal Inputs	35
3.1	Definitions	35
3.2	Implications of Singularity on the Admissible Region	38
3.2.1	Admissible Region in the Absence of State Constraints	38
3.2.2	Admissible Region in the Presence of State Constraints	39
3.3	Linear Systems	39
3.4	Nonlinear Systems	43
3.4.1	Invariant Accessible (IA) Systems	43
3.4.2	Cost-related Invariant Accessible (CIA) Systems	47
3.4.3	Bound-related Invariant Accessible (BIA) Systems	49
3.5	Examples	52
3.5.1	Single-input Invariant Accessible System	52
3.5.2	Cost-related Invariant Accessible System	55
3.5.3	Multiple-input Invariant Accessible System	59
3.6	Conclusions	61
4	Computation of Optimal Inputs	63
4.1	Parameterization via Optimal Feedback Laws	64
4.1.1	On the Bounds	64
4.1.2	On the Path Constraints	64
4.1.3	In the Interior of the Admissible Region	67
4.2	Determination of Switching Times	72
4.3	Example: Single-input IA System (cont'd)	73
4.4	Conclusions	77
5	Feedback Implementation of Optimal Profiles	78
5.1	Open-loop Optimization versus Closed-loop Implementation	78
5.2	Closed-loop Implementation via Feedback Structures	79
5.2.1	Nonlinear Feedback Laws	80
5.2.2	Setpoint Tracking	82
5.3	Example	85
5.4	Conclusions	87
6	Optimization of a Fixed-Bed Bioreactor	90
6.1	Modeling	90
6.1.1	Substrate Removal Kinetics	94
6.1.2	Gas-liquid Transfer of Oxygen	97
6.1.3	Mass Transfer of Particles	98

6.1.4	Simulation Results	99
6.2	Optimization Problem Formulation	104
6.3	Numerical Optimization	107
6.4	Characterization of Optimal Inputs	111
6.4.1	Reduced Model Formulation	112
6.4.2	Optimization Problem for the Reduced Model	113
6.4.3	Characterization of Optimal Inputs Using the Reduced Model	114
6.5	Feedback Implementation of the Optimal Solution	126
6.5.1	PI Controller	127
6.5.2	Linearizing Control Scheme	129
6.6	Conclusions	135
7	Conclusions and Outlook	137
7.1	Significance of the Developed Methodology	137
7.2	Application to a Biofilter	139
7.3	Outlook	139
	Appendix	141
A	Numerical Values	141
A.1	Example Single-input IA System	141
A.2	Biofilter	142
B	Proofs Concerning the Biofilter	144
B.1	Proof of Proposition 2	144
B.2	Proof of Proposition 3	146
B.3	Proof of Proposition 5	149
	Bibliography	151