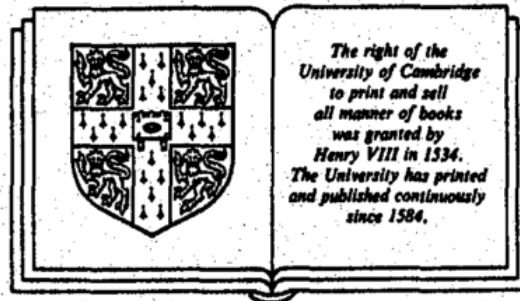


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*Heat kernels  
and spectral theory*



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