

# CONTENTS

## MODULE A APPLICABLE MATHEMATICS †

- A1 Introduction to Applicable Mathematics 1**  
What is applicable mathematics? Motivation for using computers;  
Against the abuse of computer power
- A2 Complex Numbers and Complex Exponentials 4**
- A2.1 Definitions: The algebra of complex numbers 4  
Rules for complex numbers; Computers and complex numbers;  
Complex numbers and geometry; Complex conjugation, modulus,  
argument
- A2.2 The complex plane: De Moivre's theorem 10  
Cartesian and plane-polar coordinates; De Moivre's theorem
- A2.3 Complex exponentials: Euler's theorem 13  
Deriving Euler's theorem; Applying Euler's theorem
- A2.4 Hyperbolic functions 15  
Definition of hyperbolic functions; Circular-hyperbolic analogs
- A2.5 Phase angles and vibrations 18
- A2.6 Diversion: The interpretation of complex numbers 19
- Problems with complex numbers 20  
*Finding buried treasure; Proofs of De Moivre's theorem; Exponentials and the  
FFT; Exponential fun; Graphing hyperbolic functions; Analytic continuation*
- References on complex numbers 21
- A3 Power Series Expansions 23**
- A3.1 Motivation for using power series 23

† Items in italics indicate extensive problems.

A3.2	Convergence of power series	24
	The geometric series; Geometric series summed	
A3.3	Taylor series and their interpretation	27
	Proof of Taylor's theorem; Interpreting Taylor series	
A3.4	Taylor expansions of useful functions	29
	Expansion of exponentials; Series for circular functions; Hyperbolic function expansions	
A3.5	The binomial approximation	33
	The binomial approximation derived; Interpreting the binomial approximation; Applying the binomial approximation	
A3.6	Diversion: Financial interest schemes	35
	Problems with power series	36
	<i>Complex geometric series; Proof by induction; Numerical geometric series, Series by analogy; Numerical exponentials; Logarithms in series; Euler and Maclaurin; Numerical cosines and sines; Financing solar energy</i>	
	References on power series	39
<b>A4</b>	<b>Numerical Derivatives, Integrals, and Curve Fitting</b>	<b>40</b>
A4.1	The discreteness of data	40
	Numerical mathematics; Discreteness	
A4.2	Numerical noise	41
	Round-off error; Truncation error; Unstable problems and unstable methods; Subtractive cancellation; Numerical evaluation of polynomials	
A4.3	Approximation of derivatives	47
	Forward-difference derivatives; Central-difference derivatives; Error estimates for numerical derivatives; Numerical second derivatives	
A4.4	Numerical integration	51
	Trapezoid formula; The Simpson formula; Comparing trapezoid and Simpson formulas	
A4.5	Diversion: Analytic evaluation by computer	54

A4.6 Curve fitting by splines 55  
 Introduction to splines; Derivation of spline formulas; Algorithm for spline fitting; Spline properties; Development of splines and computers

A4.7 The least-squares principle 61  
 Choice of least squares; Derivation of least-squares equations; Use of orthogonal functions; Fitting averages and straight lines

Problems in numerical analysis 68  
*Round-off and truncation noise; Numerical convergence of series; Subtractive cancellation in quadratics; Comparison of polynomial algorithms; Numerical integration algorithms; Integral approximations to series; Natural splines have minimal curvature; Program for cubic splines; Orthogonal polynomials for least-squares fit; Stability of least-squares algorithms*

References on numerical analysis 71

## A5 Fourier Expansions 73

A5.1 Overview of Fourier expansions 73  
 Transformations; Nomenclature of Fourier expansions

A5.2 Discrete Fourier transforms 75  
 Derivation of the discrete transform; Properties of the discrete transform

A5.3 Fourier series: Harmonic approximations 78  
 From discrete transforms to series; Interpreting Fourier coefficients

A5.4 Examples of Fourier series 80  
 Square pulses; Fourier coefficients and symmetry conditions; The wedge function; Window functions; Convergence of Fourier series

A5.5 Diversion: The Gibbs phenomenon 87

A5.6 Fourier series for arbitrary intervals 88  
 Interval scaling; Magnitude scaling

A5.7 Fourier integral transforms 89  
 From Fourier series to Fourier integrals; Applying Fourier integral transforms

A5.8	A fast Fourier transform algorithm	93
	Derivation of FFT algorithm; Reordering the FFT coefficients; Comparing FFT with conventional transforms	
	Problems on Fourier expansions	98
	<i>Relating Fourier coefficients through algebra; Relating Fourier coefficients through geometry; Smoothing window functions; Gibbs phenomenon and Lanczos damping factors; Dirac delta function; FFT in remote-sensing applications</i>	
	References on Fourier expansions	100
<b>A6</b>	<b>Introduction to Differential Equations</b>	<b>101</b>
	From the particular to the general	
A6.1	Differential equations and physical systems	101
	Why differential equations? Notation and classification; Homogeneous and linear equations	
A6.2	Separable differential equations	104
	Relating student scores and work; Generalizing separable equations	
A6.3	First-order linear equations: World-record sprints	107
	Kinematics of world-record sprints; Limbering up; Physics and physical activity; General first-order differential equation	
A6.4	Diversion: The logarithmic century	111
	Interpreting exponential behavior; Bell's decibels	
A6.5	Nonlinear differential equations	114
	The logistic growth curve; Exploring logistic growth	
A6.6	Numerical methods for first-order equations	118
	Predictor formulas; Initial values for solutions; Stability of numerical methods	
	Problems on first-order differential equations	123
	<i>Acceleration and speed in sprints; World-class sprinters; Attenuation in solar collectors; Predictions from exponential growth; Belles, bells and decibels; Reaping Nature's bounty; The struggle for survival; Unstable methods for differential equations</i>	
	References on differential equations	128

- A7 Second-Order Differential Equations 130**  
 Why are second-order equations so common?
- A7.1 Cables and hyperbolic functions 131  
 Getting the hang of it; Interpreting the cable parameter; Solving the cable differential equation; Exercises with catenaries
- A7.2 Diversion: History of the catenary 136
- A7.3 Second-order linear differential equations 136  
 Mechanical and electrical analogs; Solving the equations for free motion; Discussion of free-motion solutions
- A7.4 Forced motion and resonances 142  
 Differential equation with a source term; Alternative treatment by Fourier transforms; Resonant oscillations; The Lorentzian function
- A7.5 Electricity in nerve fibers 147  
 Modeling nerve fibers; Solution of the axon potential
- A7.6 Numerical methods for second-order equations 150  
 Euler approximations; Numerov's method for linear equations; Comparisons of Euler and Numerov algorithms
- A7.7 Solution of stiff differential equations 155  
 What is a stiff differential equation? The Riccati transformation
- Problems on second-order differential equations 158  
*Cables and arcs; Suspension bridges; Properties of Lorentzians; Electricity in axons; Stability of Euler approximations; Madelung transformations for stiff equations*
- References on differential equations 162
- A8 Applied Vector Dynamics 163**
- A8.1 Kinematics in Cartesian and polar coordinates 163  
 Polar coordinate unit vectors; Velocity and acceleration in polar coordinates
- A8.2 Central forces and inverse-square forces 166  
 Angular momentum conservation: Kepler's first law; Areal velocity: Kepler's second law

- A8.3 Satellite orbits 169  
Inverse-square force differential equations; Analytic solution of orbit equations; Some geometry of ellipses; Relation of period to axes: Kepler's third law
- A8.4 Diversion: Kepler's Harmony of the World 175
- A8.5 Summary of Keplerian orbits 176  
Kepler's three laws; The inverse-square laws from Kepler's laws
- Problems on vector dynamics 178  
*Earth-bound projectiles; Electrostatic-force orbits; Solar properties; Comets; Interstellar travel*
- References on vector dynamics 180

## MODULE L LABORATORIES IN COMPUTING †

- L1 Introduction to the Computing Laboratories 182**  
Analysis and simulation by computer; Coding is not programming is not computing; The programming languages; Exploring with the computer; References on languages
- L2 Conversion Between Polar and Cartesian Coordinates 185**
- L2.1 Cartesian coordinates from polar coordinates 185  
Pascal program for conversion to Cartesian coordinates; Fortran program for conversion to Cartesian coordinates; Exercises on Cartesian from polar coordinates; *Cartesian coordinates from polar coordinates*
- L2.2 Polar coordinates from Cartesian coordinates 187  
Pascal program for conversion to polar coordinates; Fortran program for conversion to polar coordinates; Exercises on polar from Cartesian coordinates; *Polar coordinates from Cartesian coordinates*
- L3 Numerical Approximation of Derivatives 192**
- L3.1 Forward and central difference methods 193  
Extrapolation to the limit

† Items in italics indicate extensive programming exercises.

- L3.2 Exercises in numerical differentiation 194  
 Numerical derivatives program structure; Exercises on numerical derivatives; *Numerical derivatives of simple functions*; *Extrapolating to the limit for derivatives*; *Second derivatives numerically estimated*; Other differentiation techniques
- References on numerical derivatives 197
- L4 An Introduction to Computer Graphics 198**  
 Why printer graphics?
- L4.1 Plotting using printers 199  
 Formulas for scales and origins; Printer plotting procedure structure
- L4.2 Sample programs for printer plots 201  
 Printer plots from Pascal; Printer plots from Fortran; Plotting exercises; *Graphic examples*, *Improving your image*; *A pot-pourri of plots*
- L4.3 Other graphics techniques 209  
 Video-screen and interactive graphics; Static graphics
- References on computer graphics 210
- L5 Electrostatic Potentials by Integration 212**
- L5.1 Analytic derivation of line-charge potential 212  
 Electrostatic potential of a line charge; On-axis potential of a line charge; Symmetries and reflections; Scaling the line-charge potential formulas
- L5.2 Line-charge potential using trapezoid formula 216  
 Structure of the electrostatic potential program; *Analytic evaluation of the potential*; *Trapezoid program for line-charge potential*
- L5.3 Potential integral from Simpson's formula 219  
 Simpson's formula exercises; *Simpson program for line-charge potential*
- L5.4 Displaying equipotential distributions 220  
 Equipotentials of a line charge; *Programming equipotentials*; Using logarithmic scales for displays

**L6 Monte-Carlo Simulations 223**

L6.1 Generating and testing pseudo-random numbers 223  
Power-residue random-number generator; Random numbers in a given range; *Program for array of pseudo-random numbers*; The random walk

L6.2 Stimulating simulations in mathematics 227  
Simulation of round-off errors; *Random numbers and arithmetic error*; Estimating integrals and areas; *Estimating  $\pi$  by Monte-Carlo simulation*

L6.3 The approach to thermodynamic equilibrium 229  
Analytic method; Entropy and the approach to equilibrium; Monte-Carlo method; *Program for approach to equilibrium*

L6.4 Simulation of nuclear radioactivity 235  
Analytic formula for radioactivity; Monte-Carlo simulation; *Simulating radioactivity*; Other Monte-Carlo simulations

Random references on Monte-Carlo methods 238

**L7 Spline Fitting and Interpolation 239**

L7.1 Sample programs for spline fitting 239  
Structure of the spline procedures; Spline fitting from Pascal; Spline fitting from Fortran

L7.2 Examples using splines 246  
Boundary condition and interpolation exercises; *Natural and other spline boundary conditions*; *Interpolation using cubic splines*; Exercises on spline derivatives; *Derivatives from cubic splines*; Spline integration exercises; *Integration by cubic splines*

References on spline fitting 249

**L8 Least-Squares Analysis of Data 250**

L8.1 Straight-line fits with errors in both variables 250  
Straight-line least squares; Least-squares formulas

L8.2 Sample programs for straight-line fitting 253  
Straight-line fitting in Pascal; Fitting to straight lines from Fortran

L8.3    Quarks, radiocarbon dating, solar cells, and warfare    257  
 Evidence for fractional charges; *Least-squares analysis of fractional charge data*; Radiocarbon dating and Egyptian antiquities; *Regression analysis of radiocarbon and historical dates*; Solar cell efficiency in space; *Degrading performance of solar cells*; Is war on the increase? *Least-squares analysis of battle deaths*

A minimal reading list on least squares    263

**L9      Fourier Analysis of an EEG    265**

L9.1    Introduction to encephalography    266  
 What is an EEG? Salmon EEG and socioeconomics;  
 The clinical record; Fourier analysis program structure

L9.2    Frequency spectrum analysis of the EEG    269  
 Coding the Fourier amplitude calculation; *Fourier transform spectrum*;  
 FFT programs in Pascal and Fortran; Predicting voltages from  
 Fourier amplitudes; *Recomposing EEGs*; *Testing Wiener-Khinchin*

L9.3    The Nyquist criterion and noise    278  
 The Nyquist criterion; *Testing the Nyquist criterion*; Effects of noise;  
*Analyzing noise effects*

L9.4    Autocorrelation analysis of the EEG    280  
 Properties of autocorrelations; Noise and autocorrelations;  
*Autocorrelation analysis program*

References on Fourier analysis and the EEG    283

**L10    Analysis of Resonance Line Widths    285**

L10.1    A brief introduction to atomic clocks    285  
 Atomic motions as clocks; Resonances and clocks

L10.2    The inversion resonance in ammonia    287  
 Microwave absorption experiments; Data for the ammonia  
 inversion resonance

L10.3    Fourier-transform analysis of a resonance    289  
 Derivation of the transform relation; Interpreting the Fourier  
 transform; Resonance analysis program structure; Exercises on  
 resonance analysis; *Resonance line width program*

L10.4	Error analysis for the Fourier transform	294
	Finite-range-of-data errors; Finite-step-size errors; <i>Error analysis for integral transforms</i>	
	References on resonances and atomic clocks	296
<b>L11</b>	<b>Space-Vehicle Orbits and Trajectories</b>	<b>298</b>
L11.1	Space vehicles, satellites and computers	298
	Uses of space vehicles; Communications satellites; Data on Earth satellites	
L11.2	Numerical methods for orbits and trajectories	301
	For observables to orbital parameters; <i>Program for orbits</i> ; Numerical integration for trajectories; Structure of the space-vehicle program; <i>Program for trajectories</i> ; Files for space-vehicle data	
L11.3	Display of satellite orbits	309
	Geometry of the orbit display; Illusory ellipses	
	References on space vehicles and satellites	310
	<b>INDEX</b>	<b>313</b>