

CONTENTS

Chapter 1 First-Order Equations

1.1	Introduction	1
1.2	First-order equations and direction fields	5
1.3	The equation $y' = f(x)$	7
1.4	Numerical methods	10
1.5	Falling bodies	12
1.6	Newton's law $F = ma$	15
1.7	Falling bodies subject to resisting forces	18
1.8	The equation $y' = q(y)$	25
1.9	Uniqueness	27
1.10	The measurement of time	31
1.11	Clepsydra	33
1.12	Carbon-14 dating	38
1.13	The separable equation $y' = f(x)q(y)$	42
1.14	Escape velocity	46
1.15	The law of gravitation	49
1.16	Other first-order equations	53
1.17	The reflector	53
1.18	The homogeneous differential equation	56
1.19	Integrating factors for first-order linear differential equations	57

Chapter 2 Second-Order Linear Equations

2.1	Introduction	60
2.2	A linear operator	62
2.3	Linearly independent functions	65
2.4	The Wronskian determinant	66
2.5	A general form for solutions of the homogeneous linear differential equation	71
2.6	The pendulum	74
2.7	Simple harmonic motion	78
2.8	Some remarks on gravity	78
2.9	Tunnels through the earth	79
2.10	The spring clock; the longitude problem	82
2.11	The coiled spring	84
2.12	Homogeneous equations with constant coefficients	86
2.13	The damped oscillator	90

2.14	Harrison's chronometer	92
2.15	Hooke's law	93
2.16	Impressed forces	95
2.17	The nonhomogeneous equation $L_c(y) = r(x)$ with p, q constant	95
2.18	Resonance	97
2.19	Examples of resonance	100
2.20	Resonance in time measurement	105
2.21	The plucked string	107
2.22	The wave equation	109
2.23	Separation of variables	112
2.24	Implications of the boundary conditions	112
2.25	Superposition, musical sounds	114
2.26	The law of gravitation implies Kepler's first two laws	116
2.27	Some remarks on linear differential equations and series	119

Chapter 3 Series Solutions

3.1	Introduction	122
3.2	Numerical sequences	122
3.3	Sequences of functions	125
3.4	Numerical series	126
3.5	Convergence tests	128
3.6	Series of function values	132
3.7	Power series	134
3.8	Taylor series and numerical methods	136
3.9	Taylor polynomials and the remainder	138
3.10	Series solutions of differential equations	141
3.11	The vibrating membrane	143
3.12	Normal symmetric modes of vibration	148
3.13	The Bessel function J_0 and numerical approximations	149
3.14	The boundary condition $z(r_0, \theta, t) = 0$	150
3.15	Normal modes of vibration, the Bessel equation and J_1	152
3.16	Bessel functions, J_m	155
3.17	Characteristic values (or eigenvalues) and characteristic functions (or eigenfunctions).	157
3.18	Vibrating membranes: experimental results	157
3.19	Some remarks on series solution methods	167
3.20	Some remarks on the history and terminology of Bessel functions	170

Chapter 4 Existence Theorems

4.1	Introduction	172
4.2	An outline of the proof	173
4.3	The equation $y' = f(x, y)$	174
4.4	The equivalence of $y' = f(x, y)$ and the integral equation $y(x) = y_0 + \int_{x_0}^x f(t, y(t)) dt$	177

4.5	Picard's successive approximations	178
4.6	Numerical and polynomial approximations	179
4.7	Uniform convergence and the Picard approximation functions	181
4.8	Bounds and closed sets	184
4.9	I_α , a common domain for the Picard functions	185
4.10	Convergence—the Lipschitz condition	187
4.11	s_λ is a solution function	190
4.12	The solution s_λ on I_α is unique	192
4.13	A statement of the existence and uniqueness theorem for $y' = f(x, y)$	193
4.14	The proof of the existence theorem for the second-order linear equation	194
4.15	Numerical methods: résumé and general discussion	199

General Reading Suggestions

Appendix A

1.	Bounded sets and functions	205
2.	Continuous functions	205
3.	Differentiable functions	205
4.	The Riemann integral $\int_a^b f(x) dx$	206
5.	A derivation of the relation $\dot{A} = \frac{1}{2}r^2\dot{\theta}$	207

Appendix B

1.	Radian measure	208
----	--------------------------	-----

Appendix C

1.	Values of Bessel functions J_0 and J_1	209
2.	Roots of the equation $J_m(x) = 0$	209

Appendix D

1.	Proof of Lemma 4.4.1	210
----	--------------------------------	-----

Answers to Selected Problems	211
---	-----

List of Symbols	251
----------------------------------	-----

Bibliography	255
-------------------------------	-----

Index	261
------------------------	-----