

DATA ENVELOPMENT ANALYSIS

A Comprehensive Text with Models,
Applications, References
and DEA-Solver Software

Second Edition

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Contents

List of Tables	xvii
List of Figures	xxi
Preface	xxv
1. GENERAL DISCUSSION	1
1.1 Introduction	1
1.2 Single Input and Single Output	2
1.3 Two Inputs and One Output Case	6
1.4 One Input and Two Outputs Case	8
1.5 Fixed and Variable Weights	12
1.6 Summary and Conclusion	13
1.7 Problem Supplement for Chapter 1	15
2. BASIC CCR MODEL	21
2.1 Introduction	21
2.2 Data	22
2.3 The CCR Model	23
2.4 From a Fractional to a Linear Program	23
2.5 Meaning of Optimal Weights	25
2.6 Explanatory Examples	25
2.6.1 Example 2.1 (1 Input and 1 Output Case)	26
2.6.2 Example 2.2 (2 Inputs and 1 Output Case)	27
2.7 Illustration of Example 2.2	30
2.8 Summary of Chapter 2	32
2.9 Selected Bibliography	33
2.10 Problem Supplement for Chapter 2	34
3. CCR MODEL AND PRODUCTION CORRESPONDENCE	41
3.1 Introduction	41
3.2 Production Possibility Set	42
3.3 The CCR Model and Dual Problem	43
3.4 The Reference Set and Improvement in Efficiency	47

3.5	Theorems on CCR-Efficiency	48
3.6	Computational Aspects of the CCR Model	50
3.6.1	Computational Procedure for the CCR Model	50
3.6.2	Data Envelopment Analysis and the Data	52
3.6.3	Determination of Weights (=Multipliers)	52
3.6.4	Reasons for Solving the CCR Model Using the Dual (DLP_o)	52
3.7	Example	53
3.8	The Output-Oriented Model	58
3.9	An Extension of the Two Phase Process in the CCR Model	60
3.10	Discretionary and Non-Discretionary Inputs	63
3.11	Summary of Chapter 3	68
3.12	Notes and Selected Bibliography	68
3.13	Related DEA-Solver Models for Chapter 3	70
3.14	Problem Supplement for Chapter 3	71
4.	ALTERNATIVE DEA MODELS	87
4.1	Introduction	87
4.2	The BCC Models	89
4.2.1	The BCC Model	91
4.2.2	The Output-oriented BCC Model	93
4.3	The Additive Model	94
4.3.1	The Basic Additive Model	94
4.3.2	Translation Invariance of the Additive Model	97
4.4	A Slacks-Based Measure of Efficiency (SBM)	99
4.4.1	Definition of SBM	100
4.4.2	Interpretation of SBM as a Product of Input and Output Inefficiencies	101
4.4.3	Solving SBM	101
4.4.4	SBM and the CCR Measure	103
4.4.5	The Dual Program of the SBM Model	104
4.4.6	Oriented SBM Models	105
4.4.7	A Weighted SBM Model	105
4.4.8	Decomposition of Inefficiency	106
4.4.9	Numerical Example (SBM)	106
4.5	A Hybrid Measure of Efficiency (Hybrid)	106
4.5.1	A Hybrid Measure	107
4.5.2	Decomposition of Inefficiency	109
4.5.3	Comparisons with the CCR and SBM Models	110
4.5.4	An Illustrative Example	111
4.6	Russell Measure Models	112
4.7	Summary of the Basic DEA Models	114
4.8	Summary of Chapter 4	116
4.9	Notes and Selected Bibliography	117
4.10	Appendix: Free Disposal Hull (FDH) Models	117
4.11	Related DEA-Solver Models for Chapter 4	119
4.12	Problem Supplement for Chapter 4	120

5. RETURNS TO SCALE	131
5.1 Introduction	131
5.2 Geometric Portrayals in DEA	134
5.3 BCC Returns to Scale	136
5.4 CCR Returns to Scale	138
5.5 Most Productive Scale Size	143
5.6 Further Considerations	147
5.7 Relaxation of the Convexity Condition	150
5.8 Decomposition of Technical Efficiency	152
5.8.1 Scale Efficiency	152
5.8.2 Mix Efficiency	154
5.8.3 An Example of Decomposition of Technical Efficiency	155
5.9 An Example of Returns to Scale Using a Bank Merger Simulation	156
5.9.1 Background	156
5.9.2 Efficiencies and Returns to Scale	156
5.9.3 The Effects of a Merger	159
5.10 Summary	162
5.11 Additive Models	162
5.12 Multiplicative Models and "Exact" Elasticity	165
5.13 Summary of Chapter 5	170
5.14 Appendix: FGL Treatment and Extensions	171
5.15 Related DEA-Solver Models for Chapter 5	172
5.16 Problem Supplement for Chapter 5	173
6. MODELS WITH RESTRICTED MULTIPLIERS	177
6.1 Introduction	177
6.2 Assurance Region Method	178
6.2.1 Formula for the Assurance Region Method	178
6.2.2 General Hospital Example	181
6.2.3 Change of Efficient Frontier by Assurance Region Method	183
6.2.4 On Determining the Lower and Upper Bounds	184
6.3 Another Assurance Region Model	185
6.4 Cone-Ratio Method	186
6.4.1 Polyhedral Convex Cone as an Admissible Region of Weights	186
6.4.2 Formula for Cone-Ratio Method	187
6.4.3 A Cone-Ratio Example	188
6.4.4 How to Choose Admissible Directions	189
6.5 An Application of the Cone-Ratio Model	189
6.6 Negative Slack Values and Their Uses	194
6.7 A Site Evaluation Study for Relocating Japanese Government Agencies out of Tokyo	196
6.7.1 Background	196
6.7.2 The Main Criteria and their Hierarchy Structure	197
6.7.3 Scores of the 10 Sites with respect to the 18 Criteria	198
6.7.4 Weights of the 18 Criteria by the 18 Council Members (Evaluators)	199
6.7.5 Decision Analyses using Averages and Medians	201

6.7.6	Decision Analyses using the Assurance Region Model	201
6.7.7	Evaluation of "Positive" of Each Site	202
6.7.8	Evaluation of "Negative" of Each Site	202
6.7.9	Uses of "Positive" and "Negative" Scores	203
6.7.10	Decision by the Council	203
6.7.11	Concluding Remarks	204
6.8	Summary of Chapter 6	205
6.9	Notes and Selected Bibliography	206
6.10	Related DEA-Solver Models for Chapter 6	206
6.11	Problem Supplement for Chapter 6	207
7.	NON-DISCRETIONARY AND CATEGORICAL VARIABLES	215
7.1	Introduction	215
7.2	Examples	217
7.3	Non-controllable, Non-discretionary and Bounded Variable Models	219
7.3.1	Non-controllable Variable (NCN) Model	219
7.3.2	An Example of a Non-Controllable Variable	220
7.3.3	Non-discretionary Variable (NDSC) Model	222
7.3.4	Bounded Variable (BND) Model	224
7.3.5	An Example of the Bounded Variable Model	224
7.4	DEA with Categorical DMUs	227
7.4.1	An Example of a Hierarchical Category	227
7.4.2	Solution to the Categorical Model	228
7.4.3	Extension of the Categorical Model	229
7.5	Comparisons of Efficiency between Different Systems	231
7.5.1	Formulation	231
7.5.2	Computation of Efficiency	232
7.5.3	Illustration of a One Input and Two Output Scenario	232
7.6	Rank-Sum Statistics and DEA	233
7.6.1	Rank-Sum-Test (Wilcoxon-Mann-Whitney)	234
7.6.2	Use of the Test for Comparing the DEA Scores of Two Groups	235
7.6.3	Use of the Test for Comparing the Efficient Frontiers of Two Groups	236
7.6.4	Bilateral Comparisons Using DEA	236
7.6.5	An Example of Bilateral Comparisons in DEA	237
7.6.6	Evaluating Efficiencies of Different Organization Forms	238
7.7	Summary of Chapter 7	240
7.8	Notes and Selected Bibliography	240
7.9	Related DEA-Solver Models for Chapter 7	240
7.10	Problem Supplement for Chapter 7	242
8.	ALLOCATION MODELS	257
8.1	Introduction	257
8.2	Overall Efficiency with Common Prices and Costs	258
8.2.1	Cost Efficiency	258
8.2.2	Revenue Efficiency	260
8.2.3	Profit Efficiency	260
8.2.4	An Example	261

8.3	New Cost Efficiency under Different Unit Prices	262
8.3.1	A New Scheme for Evaluating Cost Efficiency	262
8.3.2	Differences Between the Two Models	264
8.3.3	An Empirical Example	265
8.3.4	Extensions	267
8.4	Decomposition of Cost Efficiency	269
8.4.1	Loss due to Technical Inefficiency	269
8.4.2	Loss due to Input Price Inefficiency	270
8.4.3	Loss due to Allocative Inefficiency	271
8.4.4	Decomposition of the Actual Cost	271
8.4.5	An Example of Decomposition of Actual Cost	272
8.5	Summary of Chapter 8	272
8.6	Notes and Selected Bibliography	273
8.7	Related DEA-Solver Models for Chapter 8	274
8.8	Problem Supplement for Chapter 8	276
9.	DATA VARIATIONS	283
9.1	Introduction	283
9.2	Sensitivity Analysis	283
9.2.1	Degrees of Freedom	283
9.2.2	Algorithmic Approaches	284
9.2.3	Metric Approaches	284
9.2.4	Multiplier Model Approaches	287
9.3	Statistical Approaches	291
9.4	Chance-Constrained Programming and Satisficing in DEA	298
9.4.1	Introduction	298
9.4.2	Satisficing in DEA	298
9.4.3	Deterministic Equivalents	299
9.4.4	Stochastic Efficiency	302
9.5	Summary of Chapter 9	304
10.	SUPER-EFFICIENCY MODELS	309
10.1	Introduction	309
10.2	Radial Super-efficiency Models	310
10.3	Non-radial Super-efficiency Models	313
10.3.1	Definition of Non-radial Super-efficiency Measure	314
10.3.2	Solving Super-efficiency	315
10.3.3	Input/Output-Oriented Super-efficiency	316
10.3.4	An Example of Non-radial Super-efficiency	316
10.4	Extensions to Variable Returns-to-Scale	317
10.4.1	Radial Super-efficiency Case	317
10.4.2	Non-radial Super-efficiency Case	318
10.5	Summary of Chapter 10	319
10.6	Notes and Selected Bibliography	319
10.7	Related DEA-Solver Models for Chapter 10	319
10.8	Problem Supplement for Chapter 10	320

11. EFFICIENCY CHANGE OVER TIME	323
11.1 Introduction	323
11.2 Window Analysis	324
11.2.1 An Example	324
11.2.2 Application	324
11.2.3 Analysis	326
11.3 Malmquist Index	328
11.3.1 Dealing with Panel Data	328
11.3.2 Catch-up Effect	329
11.3.3 Frontier-shift Effect	329
11.3.4 Malmquist Index	330
11.3.5 The Radial MI	331
11.3.6 The Non-radial and Slacks-based MI	333
11.3.7 The Non-radial and Non-oriented MI	336
11.3.8 Scale Efficiency Change	337
11.3.9 Illustrative Examples for Model Comparisons	338
11.3.10 Concluding Remarks	344
11.4 Summary of Chapter 11	345
11.5 Notes and Selected Bibliography	345
11.6 Related DEA-Solver Models for Chapter 11	345
12. SCALE ELASTICITY AND CONGESTION	349
12.1 Introduction	349
12.2 Scale Elasticity in Production	350
12.3 Congestion	353
12.3.1 Strong Congestion	354
12.3.2 Weak Congestion	357
12.3.3 Summary of Degree of Scale Economies and Congestion	360
12.4 Illustrative Examples	360
12.4.1 Degree of Scale Economies and Strong Congestion	360
12.4.2 Weak vs. Strong Congestion	361
12.5 Summary of Chapter 12	362
12.6 Notes and Selected Bibliography	363
12.7 Related DEA-Solver Models for Chapter 12	364
12.8 Problem Supplement for Chapter 12	364
13. UNDESIRABLE OUTPUTS MODELS	367
13.1 Introduction	367
13.2 An SBM with Undesirable Outputs	368
13.2.1 An Undesirable Output Model	368
13.2.2 Dual Interpretations	369
13.2.3 Returns-to-scale (RTS) Issues	370
13.2.4 Imposing Weights to Inputs and/or Outputs	370
13.3 Non-separable 'Good' and 'Bad' Output Model	371
13.4 Illustrative Examples	374
13.4.1 Separable Bad Outputs Models	374
13.4.2 An Example with Both Separable and Non-separable Inputs/Outputs	375

13.5 Comparisons with Other Methods	376
13.6 Summary of Chapter 13	378
13.7 Related DEA-Solver Models for Chapter 13	378
14. ECONOMIES OF SCOPE AND CAPACITY UTILIZATION	381
14.1 Introduction	381
14.2 Economies of Scope	382
14.2.1 Definition	382
14.2.2 Checking for Economies of Scope	382
14.2.3 Checking a Virtual Merger	386
14.2.4 Comparisons of Business Models	387
14.2.5 Illustrative Examples	388
14.3 Capacity Utilization	390
14.3.1 Fixed vs. Variable Input Resources	390
14.3.2 Technical Capacity Utilization	391
14.3.3 Price-Based Capacity Utilization Measure	392
14.3.4 Long-Run and Short-Run Capacity Utilization	395
14.3.5 Illustrative Examples	396
14.4 Summary of Chapter 14	401
14.5 Notes and Selected Bibliography	401
14.6 Related DEA-Solver Models for Chapter 14	402
14.7 Problem Supplement for Chapter 14	402
15. A DEA GAME	405
15.1 Introduction	405
15.2 Formulation	406
15.3 Coalition and Characteristic Function	409
15.4 Solution	411
15.4.1 Coalition and Individual Contribution	411
15.4.2 The Shapley Value	411
15.5 DEA min Game	414
15.6 Summary of Chapter 15	415
15.7 Notes and Selected Bibliography	416
15.8 Problem Supplement for Chapter 15	416
16. MULTI-STAGE USE OF PARAMETRIC AND NON-PARAMETRIC MODELS	423
16.1 Introduction	423
16.2 OLS Regressions	423
16.3 Modification and Extensions	424
16.4 Stochastic Frontier Analysis and Composed Error Models	426
16.5 DEA and Regression Combinations	427
16.6 Multi-stage DEA-Regression Combinations and its Application to Japanese Banking	428
16.6.1 Introduction	428
16.6.2 The Multistage Framework	430
16.6.3 An Application to Japanese Banking	433

16.6.4 Discussion	438
16.6.5 Summary of This Case Study	439
16.7 Summary of Chapter 16	439
16.8 Notes and Selected Bibliography	439
Appendices	443
A– Linear Programming and Duality	443
A.1 Linear Programming and Optimal Solutions	443
A.2 Basis and Basic Solutions	443
A.3 Optimal Basic Solutions	444
A.4 Dual Problem	445
A.5 Symmetric Dual Problems	446
A.6 Complementarity Theorem	447
A.7 Farkas' Lemma and Theorem of the Alternative	448
A.8 Strong Theorem of Complementarity	449
A.9 Linear Programming and Duality in General Form	451
B– Introduction to DEA-Solver	454
B.1 Platform	454
B.2 Installation of DEA-Solver	454
B.3 Notation of DEA Models	454
B.4 Included DEA Models	456
B.5 Preparation of the Data File	456
B.5.1 The CCR, BCC, IRS, DRS, GRS, SBM, Super-Efficiency, Scale Elasticity, Congestion and FDH Models	456
B.5.2 The AR Model	457
B.5.3 The ARG Model	458
B.5.4 The NCN and NDSC Models	459
B.5.5 The BND Model	460
B.5.6 The CAT, SYS and Bilateral Models	460
B.5.7 The Cost and New-Cost Models	461
B.5.8 The Revenue and New-Revenue Models	462
B.5.9 The Profit, New-Profit and Ratio Models	462
B.5.10 The Window and Malmquist Models	462
B.5.11 The Hybrid Model	463
B.5.12 Weighted SBM Model	464
B.5.13 The Bad Outputs Model	465
B.5.14 The Non-separable Outputs Model	465
B.6 Starting DEA-Solver	466
B.7 Results	466
B.8 Data Limitations	473
B.8.1 Problem Size	473
B.8.2 Inappropriate Data for Each Model	474
B.9 Sample Problems and Results	475
B.10 Summary	475
B.10.1 Models that Require Numbers to be Supplied through Keyboard	475

B.10.2 Summary of Headings to Inputs/Outputs	475
C– Bibliography	477
Index	479
Index	483