

Translations of

# MATHEMATICAL MONOGRAPHS

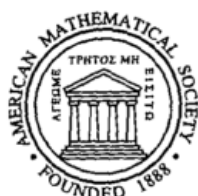
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Volume 237

## Operator Algebras and Geometry

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