

# Contents

|         |     |
|---------|-----|
| Preface | vii |
|---------|-----|

## CHAPTER I

### The Radon Transform on $\mathbf{R}^n$

|                                                                                                                        |    |
|------------------------------------------------------------------------------------------------------------------------|----|
| §1 Introduction                                                                                                        | 1  |
| §2 The Radon Transform of the Spaces $\mathcal{D}(\mathbf{R}^n)$ and $\mathcal{S}(\mathbf{R}^n)$ . The Support Theorem | 2  |
| §3 The Inversion Formula. Injectivity Questions                                                                        | 16 |
| §4 The Plancherel Formula                                                                                              | 25 |
| §5 Radon Transform of Distributions                                                                                    | 27 |
| §6 Integration over $d$ -planes. X-ray Transforms. The Range of the $d$ -plane Transform                               | 32 |
| §7 Applications                                                                                                        | 46 |
| A. Partial Differential Equations. The Wave Equation                                                                   | 46 |
| B. X-ray Reconstruction                                                                                                | 52 |
| Exercises and Further Results                                                                                          | 56 |
| Bibliographical Notes                                                                                                  | 60 |

## CHAPTER II

### A Duality in Integral Geometry

|                                                                   |     |
|-------------------------------------------------------------------|-----|
| §1 Homogeneous Spaces in Duality                                  | 63  |
| §2 The Radon Transform for the Double Fibration                   | 67  |
| (i) Principal Problems                                            | 68  |
| (ii) Ranges and Kernels. General Features                         | 71  |
| (iii) The Inversion Problem. General Remarks                      | 72  |
| §3 Orbital Integrals                                              | 75  |
| §4 Examples of Radon Transforms for Homogeneous Spaces in Duality | 77  |
| A. The Funk Transform                                             | 77  |
| B. The X-ray Transform in $\mathbf{H}^2$                          | 80  |
| C. The Horocycles in $\mathbf{H}^2$                               | 82  |
| D. The Poisson Integral as a Radon Transform                      | 86  |
| E. The $d$ -plane Transform                                       | 88  |
| F. Grassmann Manifolds                                            | 90  |
| G. Half-lines in a Half-plane                                     | 91  |
| H. Theta Series and Cusp Forms                                    | 94  |
| I. The Plane-to-Line Transform in $\mathbf{R}^3$ . The Range      | 95  |
| J. Noncompact Symmetric Space and Its Family of Horocycles        | 103 |
| Exercises and Further Results                                     | 104 |
| Bibliographical Notes                                             | 108 |

### CHAPTER III

#### The Radon Transform on Two-Point Homogeneous Spaces

|                                                                 |     |
|-----------------------------------------------------------------|-----|
| §1 Spaces of Constant Curvature. Inversion and Support Theorems | 111 |
| A. The Euclidean Case $\mathbf{R}^n$                            | 114 |
| B. The Hyperbolic Space                                         | 118 |
| C. The Spheres and the Elliptic Spaces                          | 133 |
| D. The Spherical Slice Transform                                | 145 |
| §2 Compact Two-Point Homogeneous Spaces. Applications           | 147 |
| §3 Noncompact Two-Point Homogeneous Spaces                      | 157 |
| §4 Support Theorems Relative to Horocycles                      | 159 |
| Exercises and Further Results                                   | 167 |
| Bibliographical Notes                                           | 168 |

### CHAPTER IV

#### The X-Ray Transform on a Symmetric Space

|                                                                                  |     |
|----------------------------------------------------------------------------------|-----|
| §1 Compact Symmetric Spaces. Injectivity and Local Inversion.<br>Support Theorem | 171 |
| §2 Noncompact Symmetric Spaces. Global Inversion and General<br>Support Theorem  | 178 |
| §3 Maximal Tori and Minimal Spheres in Compact Symmetric Spaces                  | 180 |
| Exercises and Further Results                                                    | 182 |
| Bibliographical Notes                                                            | 183 |

### CHAPTER V

#### Orbital Integrals and the Wave Operator for Isotropic Lorentz Spaces

|                                                                             |     |
|-----------------------------------------------------------------------------|-----|
| §1 Isotropic Spaces                                                         | 185 |
| A. The Riemannian Case                                                      | 186 |
| B. The General Pseudo-Riemannian Case                                       | 186 |
| C. The Lorentzian Case                                                      | 190 |
| §2 Orbital Integrals                                                        | 190 |
| §3 Generalized Riesz Potentials                                             | 199 |
| §4 Determination of a Function from Its Integral over Lorentzian<br>Spheres | 202 |
| §5 Orbital Integrals and Huygens' Principle                                 | 206 |
| Bibliographical Notes                                                       | 208 |

### CHAPTER VI

#### The Mean-Value Operator

|                                                |     |
|------------------------------------------------|-----|
| §1 An Injectivity Result                       | 209 |
| §2 Ásgeirsson's Mean-Value Theorem Generalized | 211 |
| §3 John's Identities                           | 215 |
| Exercises and Further Results                  | 217 |
| Bibliographical Notes                          | 219 |

## CHAPTER VII

### Fourier Transforms and Distributions. A Rapid Course

|                                                                                                                       |     |
|-----------------------------------------------------------------------------------------------------------------------|-----|
| §1 The Topology of Spaces $\mathcal{D}(\mathbf{R}^n)$ , $\mathcal{E}(\mathbf{R}^n)$ , and $\mathcal{S}(\mathbf{R}^n)$ | 221 |
| §2 Distributions                                                                                                      | 223 |
| §3 Convolutions                                                                                                       | 224 |
| §4 The Fourier Transform                                                                                              | 226 |
| §5 Differential Operators with Constant Coefficients                                                                  | 234 |
| §6 Riesz Potentials                                                                                                   | 236 |
| Exercises and Further Results                                                                                         | 248 |
| Bibliographical Notes                                                                                                 | 250 |

## CHAPTER VIII

### Lie Transformation Groups and Differential Operators

|                                                   |     |
|---------------------------------------------------|-----|
| §1 Manifolds and Lie Groups                       | 253 |
| §2 Lie Transformation Groups and Radon Transforms | 261 |

## CHAPTER IX

### Symmetric Spaces

|                                            |     |
|--------------------------------------------|-----|
| §1 Definition and Examples                 | 265 |
| §2 Symmetric Spaces of the Noncompact Type | 267 |
| §3 Symmetric Spaces of the Compact Type    | 273 |

|              |     |
|--------------|-----|
| Bibliography | 275 |
|--------------|-----|

|                        |     |
|------------------------|-----|
| Notational Conventions | 295 |
|------------------------|-----|

|                         |     |
|-------------------------|-----|
| Frequently Used Symbols | 297 |
|-------------------------|-----|

|       |     |
|-------|-----|
| Index | 299 |
|-------|-----|