

## Contents

---

|                                  |       |
|----------------------------------|-------|
| Foreword, by Nuel D. Belnap, Jr. | xiii  |
| Preface                          | xxvii |

### SEMANTICAL SECTION

|            |                                                                                                                                            |    |
|------------|--------------------------------------------------------------------------------------------------------------------------------------------|----|
| Memoir 1   | A Semantical Analysis of a General Modal Language $ML^\nu$ with an Application to an Axiomatization Problem concerning Classical Mechanics | 3  |
| N1.        | Introduction                                                                                                                               | 3  |
| PART I     | An Extensional Semantical Analysis of a General $\nu$ -sorted Modal Language $ML^\nu$                                                      | 10 |
| Chapter 1. | <u>Formation Rules for <math>ML^\nu</math> and <math>EL^\nu</math></u>                                                                     | 10 |
| N2.        | The type system                                                                                                                            | 10 |
| N3.        | The modal $\nu$ -sorted language $ML^\nu$ and the extensional language $EL^\nu$                                                            | 11 |
| N4.        | Some conventions. Church's lambda operator in $ML^\nu$ and $EL^\nu$                                                                        | 13 |
| Chapter 2. | <u>Semantics for <math>ML^\nu</math></u>                                                                                                   | 15 |
| N5.        | On possibility and how we shall deal with it                                                                                               | 15 |
| N6.        | Objects for $EL^\nu$ and QIs for $ML^\nu$ . Models and value assignments for $EL^\nu$ and $ML^\nu$ . Modal product and sum. Abbreviations  | 17 |
| N7.        | On the QIs for $ML^\nu$                                                                                                                    | 23 |
| N8.        | Equivalent QIs in a $\Gamma$ -case. Our designation rules, excluding the rule for descriptions                                             | 27 |
| N9.        | On some semantical concepts. Some fundamental formulas, considered by Carnap, which combine modalities with quantification                 | 30 |

|                   |                                                                                                                                                                                                                      |    |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| N10.              | Useful theorems on equivalent QIs                                                                                                                                                                                    | 33 |
| N11.              | The semantical rule for descriptions in $ML^\nu$                                                                                                                                                                     | 35 |
| N12.              | The $\underline{L}$ -truth in $ML^\nu$ of some logical axioms proposed for a $\nu$ -sorted modal calculus $MC^\nu$                                                                                                   | 41 |
| N13.              | On extensions of designators in $ML^\nu$ . Comparison with those for some languages by Carnap                                                                                                                        | 48 |
| N14.              | Some $\underline{L}$ -true matrices in $ML^\nu$ and some fallacies. The lambda operator in $ML^\nu$                                                                                                                  | 51 |
| <u>Chapter 3.</u> | <u>Translation in a Strong Sense of the <math>\nu</math>-sorted modal Language <math>ML^\nu</math> into the <math>(\nu + 1)</math>-sorted Extensional Language <math>EL^{\nu+1}</math></u>                           | 55 |
| N15.              | Explicit translation rules of $ML^\nu$ into $EL^{\nu+1}$                                                                                                                                                             | 55 |
| N16.              | Designation rules for $EL^{\nu+1}$ and semantical justification of the translation rules                                                                                                                             | 58 |
| PART II           | Some Useful Concepts Definable in the Modal Language $ML^\nu$ ; Applications to Questions concerning Foundations of Classical Mechanics and Everyday Life                                                            | 61 |
| N17.              | Introduction                                                                                                                                                                                                         | 61 |
| <u>Chapter 4.</u> | <u>Absolute Concepts and the Definition of Mass</u>                                                                                                                                                                  | 65 |
| N18.              | Absolute attributes. Extensional and intensional collections                                                                                                                                                         | 65 |
| N19.              | A rigorous procedure, based on $ML^\nu$ and absolute concepts, for defining physical magnitudes of mass points in accordance with Mach's definition of mass. Natural absolute concepts of real number and mass point | 69 |
| N20.              | Why certain absolute concepts were used in the preceding definition                                                                                                                                                  | 73 |
| N21.              | Certain disadvantages of existing extensional theories dealing with foundations of mechanics according to Mach, Kirchoff, and Painlevé                                                                               | 81 |
| N22.              | Usefulness of basing foundations of mechanics according to Painlevé's ideas on the modal language $ML^\nu$                                                                                                           | 84 |
| <u>Chapter 5.</u> | <u>Absolute Attributes in Connection with a Double Use of Common Nouns, with Logic, and with Natural Numbers</u>                                                                                                     | 86 |
| N23.              | A double use of common nouns in physics. On substances and qualities                                                                                                                                                 | 86 |

|                   |                                                                                                                            |     |
|-------------------|----------------------------------------------------------------------------------------------------------------------------|-----|
| N24.              | Extension of the double use of common nouns. Quasi-absolute concepts                                                       | 91  |
| N25.              | A new admissible modal axiom                                                                                               | 94  |
| N26.              | On the concept of closure in $ML^\nu$                                                                                      | 97  |
| N27.              | A natural absolute concept of natural number defined on purely logical grounds in $ML^\nu$ .<br>Peano's axioms in $ML^\nu$ | 99  |
| Notes to Memoir 1 |                                                                                                                            | 103 |

## SYNTACTICAL SECTION

|                   |                                                                                                                                                             |            |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|
| <b>Memoir 2</b>   | <b>A Modal <math>\nu</math>-sorted Logical Calculus <math>MC^\nu</math> Valid in the General Modal Language <math>ML^\nu</math></b>                         | <b>119</b> |
| N28.              | Introduction                                                                                                                                                | 119        |
| <u>Chapter 6.</u> | <u>Reduction of Part of the Modal Calculus <math>MC^\nu</math> to Extensional Logic</u>                                                                     | <u>124</u> |
| N29.              | Deduction in the modal $\nu$ -sorted calculus $MC^\nu$                                                                                                      | 124        |
| N30.              | On the extensional translations of the axioms of the modal calculus $MC^\nu$                                                                                | 127        |
| N31.              | Invariance properties of the entailment relation for the extensional translation. Some consequences within the lower predicate calculus                     | 130        |
| N32.              | The generalization, duality, equivalence, and replacement theorems in $MC^\nu$                                                                              | 134        |
| N33.              | Theorems for shortening deductions in $MC^\nu$ .<br>Rules <u>G</u> and <u>C</u>                                                                             | 139        |
| N34.              | Axioms and basic theorems for identity in $MC^\nu$                                                                                                          | 145        |
| <u>Chapter 7.</u> | <u>Modal Theorems on Extensional Matrices, Identity, and Descriptions</u>                                                                                   | <u>147</u> |
| N35.              | On extensional matrices in $MC^\nu$ . How to reduce theorems on identity in $MC^\nu$ for these matrices to theorems in extensional logic                    | 147        |
| N36.              | Theorems on identity in $MC^\nu$ for matrices of any kind                                                                                                   | 151        |
| N37.              | Some theorems in $MC^\nu$ on $(\exists \underline{x})$ , $(\exists_1 \underline{x})$ , and $(\exists_1^? \underline{x})$ that have no extensional analogues | 153        |
| N38.              | On descriptions in $MC^\nu$                                                                                                                                 | 154        |
| N39.              | Some further theorems based on our axioms for descriptions                                                                                                  | 161        |
| <u>Chapter 8.</u> | <u>On Attributes, Functions, and Natural Numbers</u>                                                                                                        | <u>165</u> |
| N40.              | Axioms for attributes and functions. First theorems                                                                                                         | 165        |

|                    |                                                                                                                                                         |     |
|--------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| N41.               | Some properties of the attributes <u>MConst</u> , <u>MSep</u> , and <u>Abs</u>                                                                          | 168 |
| N42.               | General theorems on equivalence and substitution                                                                                                        | 171 |
| N43.               | General theorems on extensionalization. An extensional form of the equivalence theorem in $MC^\nu$                                                      | 174 |
| N44.               | On the concept of closure in $MC^\nu$                                                                                                                   | 179 |
| N45.               | On natural numbers in $MC^\nu$                                                                                                                          | 183 |
| Notes to Memoir 2  |                                                                                                                                                         | 187 |
| <b>Memoir 3</b>    | <b>Elementary Possible Cases, Intensional Descriptions, and Completeness in the Modal Calculus <math>MC^\nu</math></b>                                  | 193 |
| N46.               | Introduction                                                                                                                                            | 193 |
| <u>Chapter 9.</u>  | <u>Definition in <math>MC^\nu</math> of Some Analogues of the <math>\Gamma</math>-cases and Their Occurrence</u>                                        | 197 |
| N47.               | Definition in $ML^\nu$ of the first analogue <u>EIR</u> of the concept of elementary possible case                                                      | 197 |
| N48.               | The second analogue <u>EI</u> of the concept of elementary possible case                                                                                | 202 |
| N49.               | Characterizations of <u>Np</u> and $\diamond p$ using <u>EIR</u> and <u>EI</u> . A property of <u>EI</u>                                                | 204 |
| <u>Chapter 10.</u> | <u>A Definition in <math>MC^\nu</math> of Some Description Operators; Formal Treatment of the Real Elementary Case by Means of a Primitive Constant</u> | 210 |
| N50.               | Definition in $MC^\nu$ of the intensional description operator $\iota_u$                                                                                | 210 |
| N51.               | Definition in $MC^\nu$ of the combination $\gamma_u$ of $\iota_u$ and $\gamma$                                                                          | 212 |
| N52.               | An axiom that introduces a constant representing the real elementary case. The calculus $MC^\nu_\rho$ and the modal operator $\mathfrak{R}$             | 215 |
| N53.               | Semantics for $MC^\nu_\rho$ . Comparison of $MC^\nu_\rho$ with a theory by Prior and Meredith                                                           | 217 |
| N54.               | Discussion of $\iota_u$ and $\gamma_u$                                                                                                                  | 221 |
| N55.               | Solution of some philosophical puzzles by means of $\gamma$ , $\iota_\rho$ , and $\iota_u$ . Comparison with some solutions by Thomason and Stalnaker   | 223 |

|                    |                                                                                                                                                                                                                                            |     |
|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| <u>Chapter 11.</u> | <u>Definition within <math>MC^\nu</math> of Certain Analogues of the QIs for <math>ML^\nu</math>; the Translation <math>\Delta \rightarrow \Delta^*</math> of <math>EL^\nu</math> and <math>EL^{\nu+1}</math> into <math>ML^\nu</math></u> | 230 |
| N56.               | A set of absolute concepts in $MC^\nu$ that can characterize the object system used in our semantical theory for $EL^{\nu+1}$                                                                                                              | 230 |
| N57.               | The translation $\Delta \rightarrow \Delta^*$ of $EL^\nu$ and $EL^{\nu+1}$ into $ML^\nu$                                                                                                                                                   | 235 |
| N58.               | Invariance of the entailment relation for the translation $\Delta \rightarrow \Delta^*$ of $EL^{\nu+1}$ into $ML^\nu$                                                                                                                      | 239 |
| <u>Chapter 12.</u> | <u>A Semantical System for <math>ML^\nu</math> Defined in <math>ML^\nu</math> Itself; Invariance of the Entailment Relation under the Extensional Translation <math>\Delta \rightarrow \Delta^\eta</math> in Both Senses</u>               | 247 |
| N59.               | An assignment of analogues of <u>QIs</u> made within $MC^\nu$                                                                                                                                                                              | 247 |
| N60.               | Basic properties of $\Delta^{\eta*}$ for $\Delta$ in certain classes of designators in $ML^\nu$                                                                                                                                            | 256 |
| N61.               | The analogue $\frac{*t}{u}$ of $\frac{t}{\gamma}$ within $ML^\nu$                                                                                                                                                                          | 259 |
| N62.               | A basic conditioned equivalence between every designator $\Delta$ in $MC^\nu$ and $\Delta^{\eta*}$                                                                                                                                         | 261 |
| N63.               | Invariance of the entailment relation for the extensional translation of $MC^\nu$ into $EC^\nu$ , in both senses                                                                                                                           | 267 |
| N64.               | Relative completeness for $MC^\nu$                                                                                                                                                                                                         | 269 |
| <u>Chapter 13.</u> | <u>The Nonrival Character of Various Modal Systems According to a Suggestion by Lemmon</u>                                                                                                                                                 | 272 |
| N65.               | Introduction                                                                                                                                                                                                                               | 272 |
| N66.               | Definition in $ML^\nu$ of certain classes of conceivability properties that are possible analogues for various kinds of Kripke's acceptability relations                                                                                   | 274 |
| N67.               | The necessity operator $\underline{L}_\alpha$ corresponding to the conceivability property $\alpha$ . Cases connected with Fey's theory $\underline{T}$ and $S4$                                                                           | 275 |
| N68.               | The $\alpha$ -possibility operator $\underline{M}_\alpha$ . $\underline{L}_\alpha$ in connection with the Brouwerian system $\underline{B}$ and $S5$                                                                                       | 278 |
| N69.               | On the Barcan formula and its converse                                                                                                                                                                                                     | 280 |
| N70.               | Conclusions about our way of carrying out Lemmon's suggestions. Hints on identity, descriptions, and classes in connection with $\underline{L}_\alpha$                                                                                     | 285 |

|                                                                                                                                                    |     |
|----------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Notes to Memoir 3                                                                                                                                  | 287 |
| Appendix A A Semantical Invariance Property of Logical Constants in $ML^\nu$                                                                       | 291 |
| Appendix B A Generalization of Theor. 19.1                                                                                                         | 293 |
| Appendix C Some Requirements on the Concepts <u>Real</u> and <u>MP</u> in Connection with the Definition of Mass; Consequence concerning <u>MP</u> | 297 |
| Appendix D A Property of <u>Real</u> Following from the Preceding Requirements                                                                     | 302 |
| Appendix E On R. Montague's Work, "Pragmatics and Intensional Logic"; General operators                                                            | 304 |
| NE1. Introduction                                                                                                                                  | 304 |
| NE2. On the set theoretical techniques in $[M]$ and $[B]$                                                                                          | 307 |
| NE3. On the intensional language $IL_M$ considered in $[M]$                                                                                        | 308 |
| NE4. Operators of a general kind for $MC^\nu$                                                                                                      | 309 |
| Bibliography                                                                                                                                       | 315 |
| List of Symbols                                                                                                                                    | 319 |
| Index                                                                                                                                              | 323 |