

Contents

1	Introduction	1
2	Quadratic Stochastic Operators and Their Dynamics	7
2.1	Quadratic Stochastic Operators	7
2.2	One-dimensional q.s.o.s	15
2.3	Q.s.o.s and Markov Processes.....	20
2.4	Asymptotic Stability of q.s.o.s and Markov Processes	25
2.5	Comments and References	29
3	Quadratic Stochastic Processes	31
3.1	Definition of Quadratic Processes	31
3.2	Examples of Quadratic Stochastic Processes.....	37
3.3	Marginal Markov Processes Related to q.s.p.s	40
3.4	Quadratic Stochastic Operators and Discrete Time q.s.p.s	46
3.5	Construction of Quadratic Stochastic Processes	50
3.6	Weak Ergodicity of Quadratic Stochastic Processes	54
3.7	Comments and References	58
4	Analytic Methods in the Theory of Quadratic Stochastic Processes ...	61
4.1	Quadratic Processes with a Finite Set of States	61
4.2	Quadratic Processes with a Continuous Set of States.....	66
4.3	Averaging of Quadratic Stochastic Processes	75
4.3.1	The Set E Is Finite.....	76
4.3.2	The Set E Is a Continuum.....	81
4.4	Diffusion Quadratic Processes	82
4.5	Comments and References	84
5	Quantum Quadratic Stochastic Operators	85
5.1	Markov Operators	85
5.2	Quantum Quadratic Stochastic Operators	90
5.3	Quantum Markov Chains and q.q.s.o.s	96
5.4	Comments and References	100

6	Quantum Quadratic Stochastic Operators on $M_2(\mathbb{C})$	103
6.1	Description of Quantum Quadratic Stochastic Operators on $M_2(\mathbb{C})$	103
6.2	Simple Kadison–Schwarz Type q.q.s.o.s	109
6.3	Non-Simple Kadison–Schwarz Type q.q.s.o.s	117
6.4	An Example of a Non-Simple q.q.s.o. Which Is Not Kadison–Schwarz.....	122
6.5	The Dynamics of the Quadratic Operator Associated with Δ_ε	126
6.6	Stability of the Dynamics of Non-Simple q.q.s.o.s	133
6.7	Example 1	138
6.8	Example 2	142
6.9	Comments and References	145
7	Infinite-Dimensional Quadratic Operators	147
7.1	Infinite-Dimensional Quadratic Stochastic Operators	147
7.2	Volterra Operators	150
7.3	The Set of Volterra Operators	154
7.4	The Limit Behavior of Volterra Operators.....	157
7.5	Extension of Finite-Dimensional Volterra Operators	163
7.6	Comments and References	170
8	Quantum Quadratic Stochastic Processes and Their Ergodic Properties	173
8.1	Quantum Quadratic Stochastic Processes	173
8.2	Analytic Methods for q.q.s.p.s	178
8.3	The Ergodic Principle	181
8.4	Regularity of q.q.s.p.s	188
8.5	Expansion of Quantum Quadratic Stochastic Processes	192
8.6	The Connection Between the Fibrewise Markov Process and the Ergodic Principle.....	195
8.7	Conjugate Quantum Quadratic Stochastic Processes	201
8.8	Quantum Quadratic Stochastic Processes and Related Markov Processes.....	204
	8.8.1 Q.q.s.p.s of Type (A)	205
	8.8.2 Q.q.s.p.s of Type (B)	208
8.9	Tensor Products of q.s.p.s and q.q.s.p.s.....	210
8.10	Comments and References	215
	References	217
	Index	229