

Contents

PREFACE xv

TO THE TEACHER xvii

TO THE STUDENT xxi

1 POSSIBLE WORLDS 1

1. THIS AND OTHER POSSIBLE WORLDS 1
 - The realm of possibilities* 1
 - What are the limits to the possible?* 2
 - Possibility is not the same as conceivability* 3
 - Possible worlds: actual and non-actual* 4
 - Logical possibility distinguished from other kinds* 6
 - The constituents of possible worlds* 7
2. PROPOSITIONS, TRUTH, AND FALSITY 9
 - Truth and falsity defined* 9
 - Truth in a possible world* 11
 - Truth in the actual world* 12
 - The myth of degrees of truth* 12
3. PROPERTIES OF PROPOSITIONS 13
 - Possibly true propositions* 13
 - Possibly false propositions* 13
 - Contingent propositions* 14
 - Contradictories of propositions* 14
 - Noncontingent propositions* 15
 - Necessarily true propositions* 16
 - Necessarily false propositions* 17
 - More about contradictory propositions* 18
 - Some main kinds of noncontingent propositions* 19
 - Summary* 24
 - Symbolization* 25
4. RELATIONS BETWEEN PROPOSITIONS 28
 - Inconsistency* 28
 - Consistency* 30
 - Implication* 31
 - Equivalence* 35
 - Symbolization* 41

5. SETS OF PROPOSITIONS 42
Truth-values of proposition-sets 42
Modal properties of proposition-sets 42
Modal relations between proposition-sets 44
Minding our "P's and "Q's 47
6. MODAL PROPERTIES AND RELATIONS PICTURED ON
 WORLDS-DIAGRAMS 48
Worlds-diagrams for modal properties 49
Worlds-diagrams for modal relations 50
Interpretation of worlds-diagrams 50
A note on history and nomenclature 53
Capsule descriptions of modal relations 54
Appendix to section 6 57
7. IS A SINGLE THEORY OF TRUTH ADEQUATE FOR BOTH
 CONTINGENT AND NONCONTINGENT PROPOSITIONS? 58
8. THE "POSSIBLE WORLDS" IDIOM 62

2 PROPOSITIONS 65

1. INTRODUCTION 65
2. THE BEARERS OF TRUTH-VALUES 65
Thesis 1: Such things as beliefs, statements, assertions, remarks, hypotheses, and theories are the bearers of truth and falsity. 68
Thesis 2: Acts of believing (stating, asserting, etc.) are the bearers of truth-values. 68
Thesis 3: That which is believed, stated, etc., is what is true or false. 71
Thesis 4: Sentences are the bearers of truth-values. 71
Thesis 5: Sentence-tokens are the bearers of truth-values. 73
Thesis 6: Sentence-types are the bearers of truth-values. 74
Thesis 7: Context-free sentences are the bearers of truth-values. 75
Thesis 8: Context-free sentence-tokens are those things to which truth and falsity may be attributed. 76
Thesis 9: Context-free sentence-types are those things to which truth and falsity may be attributed. 76
Thesis 10: Propositions are those things to which truth and falsity may be attributed. 79
Thesis 11: Propositions are to be identified with the meanings of sentences. 80
Thesis 12: Propositions are to be identified with sets of possible worlds. 82

- Thesis 13: Propositions are abstract entities in their own right; that is, they are sui generis, they are not to be identified with any other kind of abstract entity.* 84
Categorial differences between sentences and propositions 86
One final note 86
3. THE STRUCTURE OF PROPOSITIONS: A SPECULATIVE THEORY 87
 - Concepts* 87
 - Attributes of concepts* 90
 - Identity conditions for concepts* 92
 - Analysis of propositions* 94
 - Identity conditions for propositions* 96
 4. ON REFERRING TO SENTENCES AND TO PROPOSITIONS 97
 - Techniques for referring to sentences* 97
 - Basic techniques for referring to propositions* 98
 - Advanced technique for referring to propositions: context-free references* 100
 - Untensed verbs in context-free references* 103
 5. THE OMNITEMPORALITY OF TRUTH 104
 6. PROPOSITIONS, SENTENCES, AND POSSIBLE WORLDS 108
 - The uni-linguo proviso* 110
 - The linguo-centric proviso* 111
 - Securing reference to propositions* 111
 7. SENTENTIAL AMBIGUITY AND POSSIBLE-WORLDS TESTING 113
 - Sentential ambiguity* 113
 - The method of possible-worlds testing* 114
 - Janus-faced sentences* 119
 8. POSSIBLE-WORLDS PARABLES 121
 - Case Study 1: The thesis that persons (creatures) who lack a language cannot have reflective beliefs* 122
 - Case Study 2: The thesis that persons (creatures) who lack a language cannot believe necessary truths* 125
 - Case Study 3: The thesis that a justified belief in a true proposition constitutes knowledge* 126

3 KNOWLEDGE 129

1. THE SUBJECT MATTER AND THE SCIENCE OF LOGIC 129
2. THE NATURE OF KNOWLEDGE 130

1. *Is it a necessary condition of the truth of a's knowing that P, that P should be true?* 131
 2. *Is it a necessary condition of a's knowing that P, that a believe that P?* 133
 3. *Is it a necessary condition of a's knowing that P, that a be justified in believing that P?* 136
 4. *What might the missing fourth necessary condition for a's knowing that P be?* 137
3. THE LIMITS OF HUMAN KNOWLEDGE 139
- The known and the unknown* 139
 - The knowable and the unknowable* 140
4. EXPERIENTIAL AND RATIOCINATIVE KNOWLEDGE 142
- Experiential knowledge* 142
 - Ratiocinative knowledge* 144
 - Appendix to section 4* 149
5. EMPIRICAL AND A PRIORI KNOWLEDGE 149
- Definitions of "empirical" and "a priori"* 150
 - The non-exhaustiveness and non-exclusiveness of the experiential/ratiocinative distinction* 151
 - The exhaustiveness and exclusiveness of the empirical/a priori distinction* 152
 - Is a priori knowledge certain?* 155
6. EPISTEMIC AND MODAL STATUS CONSIDERED TOGETHER 156
1. *Are there any contingent propositions which are knowable empirically?* 157
 2. *Are there any contingent propositions which are knowable both experientially and ratiocinatively?* 158
 3. *Are there any contingent propositions which are knowable ratiocinatively but which are not knowable experientially?* 163
 4. *Are there any contingent propositions which are knowable by other than experiential or ratiocinative means?* 164
 5. *Are there any contingent propositions which are unknowable?* 167
 6. *Are there any noncontingent propositions knowable empirically?* 168
 7. *Are there any noncontingent propositions which are knowable both experientially and ratiocinatively?* 170
 8. *Are there any noncontingent propositions which are knowable ratiocinatively but which are not knowable experientially?* 170

9. *Are there any noncontingent propositions which are knowable a priori but by means other than ratiocination?* 171
10. *Are there any noncontingent propositions which are unknowable?* 172
- Appendix to section 6: a complete classificatory scheme for the epistemic and modal distinctions* 174

4 THE SCIENCE OF LOGIC: AN OVERVIEW 179

1. INTRODUCTION 179
2. THE METHOD OF ANALYSIS 180
The objects of philosophical analysis 180
Three levels of analysis 181
The idea of a complete analysis 183
The need for a further kind of analysis 184
Possible-worlds analysis 185
Degrees of analytical knowledge 187
3. THE PARADOX OF ANALYSIS 189
Moore's problem 189
A Moorean solution 190
4. THE METHOD OF INFERENCE 192
The nature of inference 193
Valid and invalid propositional inferences 195
Determining the validity of inferences: the problem of justification 196
Rules of inference 198
What kind of rule is a rule of inference? 200
Inference and the expansion of knowledge 201
5. INFERENCE WITHIN THE SCIENCE OF LOGIC 205
Inference within axiomatic systems: the example of S5 205
Inference within natural deduction systems 210
The theoretical warrant of the method of direct proof 215
6. A PHILOSOPHICAL PERSPECTIVE ON LOGIC AS A WHOLE 218
The indispensability of modal concepts within propositional logics 218
Problems about the reduction principles 220
Problems about the paradoxes 224
Relevance logics 228
The move to predicate logic 230
Traditional syllogistic 232

<i>Modern predicate logic</i>	233
<i>Modal notions in predicate logic</i>	236
<i>Modalities de dicto and de re</i>	237
<i>Heterogeneous and homogeneous possible worlds</i>	239
<i>Is there really a logic of concepts?</i>	240

5 TRUTH-FUNCTIONAL PROPOSITIONAL LOGIC 247

1. INTRODUCTION 247
2. TRUTH-FUNCTIONAL OPERATORS 247
 - The uses of "not" and "it is not the case that"* 249
 - The uses of "and"* 252
 - The uses of "or"* 257
 - Interlude: compound sentences containing two or more sentential operators* 261
 - The uses of "if... then..."* 263
 - The uses of "if and only if"* 269
 - Appendix: truth-tables for wffs containing three or more letters* 272
3. EVALUATING COMPOUND SENTENCES 273
 - A note on two senses of "determined"* 277
4. ELEMENTARY TRUTH-TABLE TECHNIQUES FOR REVEALING MODAL STATUS AND MODAL RELATIONS 279
 - Modal status* 279
 - Modal relations* 284
 - Deductive validity* 290
5. ADVANCED TRUTH-TABLE TECHNIQUES 294
 - Corrected truth-tables* 294
 - Reduced truth-tables* 297
6. THE CONCEPT OF FORM 301
 - Sentences and sentential forms in a logic* 301
 - The relationship between sentences and sentence-forms* 302
7. EVALUATING SENTENCE-FORMS 306
 - The validity of sentence-forms* 306
 - Modal relations* 308
 - Implication* 308
 - Equivalence* 309
 - Inconsistency* 309
 - Argument-forms and deductive validity* 310
8. FORM IN A NATURAL LANGUAGE 311

9. WORLDS-DIAGRAMS AS A DECISION PROCEDURE FOR TRUTH-FUNCTIONAL PROPOSITIONAL LOGIC 313
10. A SHORTCUT FORMAL METHOD: REDUCTIO AD ABSURDUM TESTS 315
Summary 320

6 MODAL PROPOSITIONAL LOGIC 323

1. INTRODUCTION 323
2. MODAL OPERATORS 323
Non-truth-functionality 323
Modal and nonmodal propositions; modalized and non-modalized formulae 324
The interdefinability of the monadic and dyadic modal operators 327
3. SOME PROBLEMATIC USES OF MODAL EXPRESSIONS 329
"It is possible that" 329
Problems with the use of "it is necessary that"; the modal fallacy; absolute and relative necessity 330
4. THE MODAL STATUS OF MODAL PROPOSITIONS 333
5. THE OPERATOR "IT IS CONTINGENTLY TRUE THAT" 337
6. ESSENTIAL PROPERTIES OF RELATIONS 339
7. TWO CASE STUDIES IN MODAL RELATIONS: a light-hearted interlude 345
Case study 1: the pragmatics of telling the truth 345
Case study 2: an invalid inference and an unwitting impossible description 347
8. USING WORLDS-DIAGRAMS TO ASCERTAIN THE VALIDITY OF MODALIZED FORMULAE 350
Applications 351
The validity of the axioms of S5 356
The nonvalidity of the axiom set for S6 358
9. A SHORTCUT FORMAL METHOD FOR DETERMINING THE VALIDITY OF MODALIZED FORMULAE: modal reductios 359
10. THE NUMBER OF FORMALLY NON-EQUIVALENT SENTENCE-FORMS CONSTRUCTIBLE ON N SENTENCE-VARIABLES 365

11.	LOOKING BEYOND MODAL LOGIC TO INDUCTIVE LOGIC	370
	<i>The cardinality of a class and other concepts of class size</i>	371
	<i>The concept of contingent content</i>	372
	<i>Monadic modal functors</i>	375
	<i>What are the prospects for a fully-developed inductive logic?</i>	379
	<i>The concept of probabilification</i>	381
	<i>A dyadic modal functor for the concept of probabilification</i>	382
	INDEX	385