

Contents

| | |
|--|------|
| FOREWORD | xiii |
| AUTHORS' PREFACE | xv |
| CHAPTER | |
| I A TIME-DOMAIN ANALYSIS OF DETERMINISTIC OPTIMAL CONTROL PROBLEMS | 1 |
| 1.1. Introduction | 1 |
| 1.2. Finite Time Linear Optimal Regulators and Servomechanisms | 2 |
| 1.2.1 System models | 2 |
| 1.2.2 Adjoint system model | 12 |
| 1.2.3 The regulator and servomechanism problems | 18 |
| 1.2.4 Quadratic cost indices | 20 |
| 1.2.5 The necessary and sufficient condition for optimality | 25 |
| 1.3. Open-Loop Optimal Control and Cost Function Evaluation | 30 |
| 1.3.1 Gradient generation by perturbations | 31 |
| 1.3.2 Numerical optimization algorithms | 35 |
| 1.4. Closed-Loop Optimal Control and Gradient Optimality Condition | 39 |
| 1.4.1 Dual system relationships | 39 |
| 1.4.2 Optimal feedback realization and the matrix Riccati equation | 42 |
| 1.4.3 Time-domain spectral factorization | 50 |
| 1.5. Extensions to the Infinite Time-Domain | 55 |
| 1.5.1 State-space system concepts | 57 |
| 1.5.2 The optimal regulator problem on the infinite time domain | 60 |
| 1.6. Causality, Hilbert Space and Linear Quadratic Cost Optimization | 63 |
| 1.6.1 A Hilbert resolution space | 64 |
| 1.6.2 Causality | 66 |
| 1.6.3 Causality and the LQ optimization problem | 67 |
| 1.7. Conclusions | 74 |
| 1.8. Problems | 74 |
| 1.9. References | 80 |

CHAPTER

2 FREQUENCY-DOMAIN ANALYSIS OF DETERMINISTIC OPTIMAL CONTROL

| | |
|---|-----|
| PROBLEMS: A WIENER-HOPF APPROACH | 83 |
| 2.1. Introduction | 83 |
| 2.2. Frequency-Domain System Concepts | 84 |
| 2.2.1 Poles and zeros | 85 |
| 2.2.2 Return difference matrices | 87 |
| 2.3. Infinite Time-Domain Optimal Linear Regulators and Servomechanisms: s-Domain Solution | 90 |
| 2.3.1 Time- and frequency-domain functions | 91 |
| 2.3.2 On spectral factors | 95 |
| 2.3.3 Frequency-domain open-loop optimal control | 98 |
| 2.3.4 Optimal closed-loop controllers | 102 |
| 2.3.5 Optimal linear regulators and servomechanisms: A summary of frequency-domain results | 105 |
| 2.4. Optimal Control for Unstable Systems, Step Disturbance Functions and Integral Control | 108 |
| 2.4.1 Unstable systems | 108 |
| 2.4.2 The servomechanism problem and step disturbance functions | 114 |
| 2.4.3 A d.c.-machine control system | 119 |
| 2.4.4 Integral controllers | 125 |
| 2.5. Finite-Time Optimal Linear Regulators and Servomechanisms: s-Domain Solution | 128 |
| 2.5.1 Embedding the finite time problem in an infinite time- domain | 129 |
| 2.5.2 An unconstrained time domain gradient | 131 |
| 2.5.3 A frequency-domain solution | 134 |
| 2.5.4 The optimal system response | 137 |
| 2.5.5 A closed-loop optimal solution | 140 |
| 2.6. Frequency-Domain Characteristics of Optimality and Stability for State-feedback Optimal Controllers | 144 |
| 2.6.1 The optimal return difference relationship and system stability | 145 |
| 2.6.2 Frequency-domain interpretations | 147 |
| 2.6.3 Robustness and sensitivity | 151 |
| 2.6.4 Optimal design: A quadratic index with a cross-product term | 155 |
| 2.7. Conclusions | 164 |
| 2.8. Problems | 165 |
| 2.9. References | 167 |

CHAPTER

| | |
|---|-----|
| 3 THE ASYMPTOTIC BEHAVIOUR OF OPTIMAL ROOT LOCI | 171 |
| 3.1. Introduction | 171 |
| 3.2. Preliminaries for Asymptotic Optimal Root Loci Analysis | 173 |
| 3.2.1 Optimal and non-optimal root-loci terminology | 174 |

| | | |
|---------|---|-----|
| 3.2.2 | Symmetric and idempotent matrices | 178 |
| 3.2.3 | Finite zeros, infinite zeros and the Smith–McMillan form | 180 |
| 3.2.4 | Closed-loop eigenstructure and input direction vectors .. | 187 |
| 3.2.5 | Optimal return difference–input vector relationships ... | 190 |
| 3.3. | Optimal Root Loci: Starting and End Points | 191 |
| 3.3.1 | Starting points of the optimal root loci ($\rho \rightarrow \infty$) | 192 |
| 3.3.2 | The asymptotically finite terminal points for the optimal root loci ($\rho \rightarrow 0$) | 196 |
| 3.3.3 | The asymptotically infinite terminal points for the optimal root loci ($\rho \rightarrow 0$) | 201 |
| 3.4. | The Asymptotes for the Unbounded Optimal Closed-Loop Poles ($\rho \rightarrow 0$) | 206 |
| 3.4.1 | A reduction procedure for the optimal return difference relationship | 206 |
| 3.4.2 | Some algorithmic aspects of asymptote determination .. | 214 |
| 3.4.3 | A pivot analysis for the asymptotes of the unbounded optimal closed-loop poles | 226 |
| 3.4.4 | An algorithm for the determination of the asymptotes of the unbounded optimal closed-loop poles | 233 |
| 3.4.5 | Finite zeros, infinite zeros and cost-function-weighting invariance. | 243 |
| 3.5. | Conclusions | 252 |
| 3.6. | References | 253 |
| CHAPTER | | |
| 4 | THE DESIGN OF OPTIMAL CONTROL SYSTEMS BY COST WEIGHT SELECTION ... | 256 |
| 4.1. | Introduction | 256 |
| 4.2. | Heuristic Methods | 256 |
| 4.2.1 | Bryson's inverse square method | 257 |
| 4.2.2 | Weight selection to achieve traditional figures of merit .. | 260 |
| 4.3. | Optimal Eigenstructure Assignment | 265 |
| 4.3.1 | Regulator design with prescribed stability | 266 |
| 4.3.2 | Optimal modal control | 272 |
| 4.3.3 | Approximate methods of optimal eigenstructure assignment | 289 |
| 4.4. | Asymptotic Optimal Eigenstructure Assignment | 294 |
| 4.4.1 | Introduction | 294 |
| 4.4.2 | State regulator design: First-order asymptotic behaviour | 294 |
| 4.4.3 | Output regulator design: First-order asymptotic behaviour | 302 |
| 4.4.4 | State-regulator design: Higher-order asymptotic closed-loop poles | 318 |
| 4.4.5 | Output regulator design: Higher-order asymptotic closed-loop poles | 325 |
| 4.5. | Conclusions | 337 |
| 4.6. | References | 338 |

CHAPTER

| | | |
|-------|--|-----|
| 5 | THE MAXIMAL ACCURACY OF LINEAR OPTIMAL REGULATORS AND RELATED TOPICS | 341 |
| 5.1. | Introduction | 341 |
| 5.2. | A Hilbert Space Approach to Maximal Accuracy | 342 |
| 5.2.1 | The linear regulator and cheap optimal control | 342 |
| 5.2.2 | A Hilbert space pseudoinverse operator | 345 |
| 5.2.3 | An asymptotic analysis for maximal accuracy | 352 |
| 5.2.4 | Frequency domain conditions for maximal accuracy | 358 |
| 5.3. | Other Topics in Maximal Accuracy Theory | 364 |
| 5.3.1 | The bounded peaking of optimal state trajectories | 364 |
| 5.3.2 | The maximal accuracy of control structures | 375 |
| 5.3.3 | A state-space analysis for maximal accuracy | 382 |
| 5.4. | Conclusions | 396 |
| 5.5. | References | 397 |

CHAPTER

| | | |
|---------|---|-----|
| 6 | OPTIMAL CONTROL STRUCTURES | 399 |
| 6.1. | Introduction | 399 |
| 6.2. | Bounds for the Optimal Cost Value | 400 |
| 6.2.1 | Bounds derived from a Hilbert space analysis. | 401 |
| 6.2.1.1 | The method of randomized solutions | 409 |
| 6.2.1.2 | Upper bounds for the maximum eigenvalue, λ_M | 421 |
| 6.2.1.3 | An algorithm for bounds on the optimal cost value | 432 |
| 6.2.2 | Bounds derived using a matrix Riccati equation approach | 435 |
| 6.3. | Fixed Structure Optimal Feedback Control Design | 441 |
| 6.3.1 | Suboptimal gains for time-varying optimal feedback laws | 442 |
| 6.3.2 | Suboptimal fixed structure feedback laws | 443 |
| 6.3.3 | Structure selection for feedback laws | 479 |
| 6.4. | Input Deletion Controller Assessment | 481 |
| 6.4.1 | Theoretical basis for controller structure assessment | 483 |
| 6.4.2 | An algorithm for controller structure assessment | 489 |
| 6.4.3 | Tension control for a tandem cold-rolling mill | 498 |
| 6.5. | Conclusions | 501 |
| 6.6. | References | 502 |

CHAPTER

| | | |
|-------|---|-----|
| 7 | A DETERMINISTIC INDUSTRIAL CONTROL SYSTEM STUDY | 507 |
| 7.1. | Introduction | 507 |
| 7.2. | The Design of Shape-Control Systems for Steel Mills | 508 |
| 7.2.1 | A description of the mill mechanics | 510 |
| 7.2.2 | A static model for a Sendzimir mill and shape measurement | 514 |

CONTENTS

xi

7.2.3 A state-space model for the mill 517
7.2.4 A transfer-function system description 520
7.3 A Deterministic Optimal Control Design 524
7.3.1 Closed-loop optimal control analysis 525
7.3.2 Closed-loop system responses 531
7.4 A Stochastic Optimal Control Solution 534
7.4.1 Closed-loop stochastic optimal control 534
7.5 Shape Parameterization and Controller Design 540
7.5.1 Shape parameterization and the choice of control-
weighting matrices 541
7.5.2 Shape parameterization: An alternative viewpoint 545
7.6. First Intermediate Roll Control System Design 550
7.7 Conclusions 552
7.8 References 553

INDEX A1