

CONTENTS

GLOSSARY OF SYMBOLS

1	BASIC DESCRIPTIONS AND PROPERTIES	1
1.1	DETERMINISTIC VERSUS RANDOM DATA	1
1.2	CLASSIFICATIONS OF DETERMINISTIC DATA	3
1.2.1	Sinusoidal Periodic Data	3
1.2.2	Complex Periodic Data	4
1.2.3	Almost-Periodic Data	6
1.2.4	Transient Nonperiodic Data	7
1.3	CLASSIFICATIONS OF RANDOM DATA	9
1.3.1	Stationary Random Data	10
1.3.2	Ergodic Random Data	12
1.3.3	Nonstationary Random Data	12
1.3.4	Stationary Sample Records	13
1.4	ANALYSIS OF RANDOM DATA	14
1.4.1	Basic Descriptive Properties	14
1.4.2	Input/Output Relations	20
1.4.3	Error Analysis Criteria	22
1.4.4	Data Analysis Procedures	24
2	LINEAR PHYSICAL SYSTEMS	26
2.1	CONSTANT-PARAMETER LINEAR SYSTEMS	26
2.2	BASIC DYNAMIC CHARACTERISTICS	28
2.3	FREQUENCY RESPONSE FUNCTIONS	30
2.4	ILLUSTRATIONS OF FREQUENCY RESPONSE FUNCTIONS	31
2.4.1	Mechanical Systems	32
2.4.2	Electrical Systems	42
2.4.3	Other Systems	45
2.5	PRACTICAL CONSIDERATIONS	46
3	PROBABILITY FUNDAMENTALS	48
3.1	ONE RANDOM VARIABLE	48
3.1.1	Probability Distribution Functions	49
3.1.2	Expected Values	50

3.1.3	Change of Variables	52
3.1.4	Moment Generating and Characteristic Functions	54
3.1.5	Chebyshev's Inequality	56
3.2	TWO RANDOM VARIABLES	57
3.2.1	Expected Values and Correlation Coefficient	58
3.2.2	Distribution for Sum of Two Random Variables	59
3.2.3	Moment Generating and Characteristic Functions	61
3.3	GAUSSIAN (NORMAL) DISTRIBUTION	63
3.3.1	Central Limit Theorem	65
3.3.2	Joint Gaussian (Normal) Distribution	65
3.3.3	Moment Generating and Characteristic Functions	66
3.3.4	N -Dimensional Gaussian (Normal) Distribution	67
4	STATISTICAL PRINCIPLES	74
4.1	SAMPLE VALUES AND PARAMETER ESTIMATES	74
4.2	IMPORTANT PROBABILITY DISTRIBUTION FUNCTIONS	
4.2.1	Normal Distribution	77
4.2.2	Chi-Square Distribution	77
4.2.3	Student t Distribution	78
4.2.4	The F Distribution	80
4.3	SAMPLING DISTRIBUTIONS AND ILLUSTRATIONS	81
4.3.1	Distribution of Sample Mean with Known Variance	82
4.3.2	Distribution of Sample Variance	83
4.3.3	Distribution of Sample Mean with Unknown Variance	84
4.3.4	Distribution of Ratio of Two Sample Variances	84
4.4	CONFIDENCE INTERVALS	85
4.5	HYPOTHESIS TESTS	88
4.6	CHI-SQUARE GOODNESS-OF-FIT TEST	91
4.7	STATISTICAL INDEPENDENCE AND TREND TESTS	
4.7.1	Run Test	94
4.7.2	Reverse Arrangements Test	95
4.8	CORRELATION AND REGRESSION PROCEDURES	97
4.8.1	Linear Correlation Analysis	99
4.8.2	Linear Regression Analysis	99
		102
5	STATIONARY RANDOM PROCESSES	109
5.1	BASIC CONCEPTS	
5.1.1	Correlation (Covariance) Functions	109
5.1.2	Examples of Autocorrelation Functions	111
		113

5.1.3	Correlation Coefficient Functions	117
5.1.4	Cross-Correlation Function for Time Delay	118
5.2	SPECTRAL DENSITY FUNCTIONS	120
5.2.1	Spectra via Correlation Functions	120
5.2.2	Spectra via Finite Fourier Transforms	130
5.2.3	Spectra via Filtering-Squaring-Averaging	132
5.2.4	Coherence Functions	136
5.2.5	Cross-Spectrum for Time Delay	137
5.2.6	Uncertainty Relation	140
5.3	ERGODIC AND GAUSSIAN RANDOM PROCESSES	144
5.3.1	Ergodic Random Processes	144
5.3.2	Sufficient Conditions for Ergodicity	148
5.3.3	Gaussian Random Processes	149
5.3.4	Linear Transformations of Random Processes	152
5.4	DERIVATIVE RANDOM PROCESSES	154
5.4.1	Correlation Functions	154
5.4.2	Spectral Density Functions	158
5.4.3	Expected Number of Zero Crossings	159
6	SINGLE-INPUT / OUTPUT RELATIONSHIPS	164
6.1	SINGLE-INPUT / SINGLE-OUTPUT MODELS	164
6.1.1	Correlation and Spectral Relations	164
6.1.2	Ordinary Coherence Functions	172
6.1.3	Models with Extraneous Noise	176
6.1.4	Optimum Frequency Response Functions	181
6.2	SINGLE-INPUT / MULTIPLE-OUTPUT MODELS	185
6.2.1	Single-Input/Two-Output Model	185
6.2.2	Single-Input/Multiple-Output Model	187
6.2.3	Removal of Extraneous Noise	190
6.3	MULTIPLE-OUTPUT ARRAY MODELS	193
6.3.1	Array Gain	193
6.3.2	Array Directivity	195
7	MULTIPLE-INPUT / OUTPUT RELATIONSHIPS	201
7.1	MULTIPLE-INPUT / SINGLE-OUTPUT MODELS	201
7.1.1	General Relationships	202
7.1.2	General Case of Arbitrary Inputs	205
7.1.3	Special Case of Mutual Uncorrelated Inputs	206
7.2	TWO-INPUT / ONE-OUTPUT MODELS	207
7.2.1	Basic Relationships	207
7.2.2	Optimum Frequency Response Functions	211
7.2.3	Ordinary and Multiple Coherence Functions	213
7.2.4	Conditioned Spectral Density Functions	215
7.2.5	Partial Coherence Functions	221

7.3	GENERAL AND CONDITIONED MULTIPLE-INPUT MODELS	226
7.3.1	Conditioned Fourier Transforms	228
7.3.2	Conditioned Spectral Density Functions	229
7.3.3	Optimum Systems for Conditioned Inputs	230
7.3.4	Algorithm for Conditioned Spectra	233
7.3.5	Optimum Systems for Original Inputs	235
7.3.6	Partial and Multiple Coherence Functions	238
7.4	MATRIX FORMULAS FOR MULTIPLE-INPUT / MULTIPLE-OUTPUT MODELS	240
7.4.1	Multiple-Input/Multiple-Output Model	240
7.4.2	Multiple-Input/Single-Output Model	244
7.4.3	Model with Output Noise	246
7.4.4	Single-Input/Single-Output Model	248
8	STATISTICAL ERRORS IN BASIC ESTIMATES	252
8.1	DEFINITION OF ERRORS	252
8.2	MEAN AND MEAN SQUARE VALUE ESTIMATES	256
8.2.1	Mean Value Estimates	256
8.2.2	Mean Square Value Estimates	260
8.2.3	Variance Estimates	262
8.3	PROBABILITY DENSITY FUNCTION ESTIMATES	264
8.3.1	Bias of the Estimate	266
8.3.2	Variance of the Estimate	267
8.3.3	Normalized rms Error	268
8.3.4	Joint Probability Density Estimates	269
8.4	CORRELATION FUNCTION ESTIMATES	270
8.4.1	Bandwidth Limited White Noise	273
8.4.2	Noise-to-Signal Considerations	274
8.4.3	Location Estimates of Peak Correlation Values	276
8.5	AUTOSPECTRAL DENSITY FUNCTION ESTIMATES	278
8.5.1	Bias of the Estimate	280
8.5.2	Variance of the Estimate	282
8.5.3	Normalized rms Error	283
8.5.4	Estimates from Finite Fourier Transforms	283
8.6	RECORD LENGTH REQUIREMENTS	286
9	STATISTICAL ERRORS IN ADVANCED ESTIMATES	291
9.1	CROSS-SPECTRAL DENSITY FUNCTION ESTIMATES	291
9.1.1	Variance Formulas	294
9.1.2	Covariance Formulas	296
9.1.3	Phase Angle Estimates	300

9.2	SINGLE-INPUT / OUTPUT MODEL ESTIMATES	302
9.2.1	Bias in Frequency Response Function Estimates	304
9.2.2	Coherent Output Spectrum Estimates	307
9.2.3	Coherence Function Estimates	309
9.2.4	Gain Factor Estimates	312
9.2.5	Phase Factor Estimates	316
9.3	MULTIPLE-INPUT / OUTPUT MODEL ESTIMATES	317
9.3.1	Multiple Coherence Function Estimates	319
9.3.2	Multiple Coherent Output Spectrum Estimates	320
9.3.3	Single Conditioned-Input/Output Models	320
9.3.4	Partial Coherence Function Estimates	321
9.3.5	Partial Coherent Output Spectrum Estimates	321
9.3.6	Gain Factor Estimates for Conditioned Models	322
9.3.7	Phase Factor Estimates for Conditioned Models	322
10	DATA ACQUISITION AND PROCESSING	325
10.1	DATA COLLECTION	325
10.2	DATA RECORDING	329
10.2.1	Magnetization–Reproduction Procedures	330
10.2.2	Modulation–Demodulation Procedures	331
10.3	DATA PREPARATION	334
10.3.1	Sampling Theorems for Random Records	335
10.3.2	Sampling Procedures and Aliasing Errors	337
10.3.3	Quantization Errors	339
10.3.4	Analog-to-Digital Converters	341
10.4	DATA QUALIFICATION	342
10.4.1	Test for Stationarity	342
10.4.2	Test for Periodicities	345
10.4.3	Test for Normality	346
10.5	DATA ANALYSIS	348
10.5.1	Procedures for Analyzing Individual Records	348
10.5.2	Procedures for Analyzing a Set of Records	351
10.5.3	Test for Equivalence of Autospectra	355
10.5.4	Computational Considerations	357
11	DIGITAL DATA ANALYSIS	361
11.1	DATA PREPARATION	361
11.1.1	Data Standardization	362
11.1.2	Trend Removal	362
11.1.3	Digital Filtering	365
11.2	FOURIER SERIES AND FAST FOURIER TRANSFORMS	368
11.2.1	Standard Fourier Series Procedures	369
11.2.2	Fast Fourier Transforms	370
11.2.3	Cooley–Tukey Procedure	377

11.2.4	Procedures for Real-Valued Records	379
11.2.5	Further Related Formulas	381
11.2.6	Winograd Fourier Transform	383
11.3	PROBABILITY DENSITY FUNCTIONS	383
11.4	AUTOCORRELATION FUNCTIONS	385
11.4.1	Autocorrelation Estimates via Direct Computations	385
11.4.2	Autocorrelation Estimates via FFT Computations	386
11.5	AUTOSPECTRAL DENSITY FUNCTIONS	391
11.5.1	Basic Autospectra Estimation Procedures	391
11.5.2	Side-Lobe Leakage Suppression Procedures	393
11.5.3	Recommended Computational Steps	400
11.5.4	Zoom Transform Procedures	400
11.5.5	Other Spectral Analysis Procedures	403
11.6	JOINT RECORD FUNCTIONS	405
11.6.1	Joint Probability Density Functions	405
11.6.2	Cross-Correlation Functions	406
11.6.3	Cross-Spectral Density Functions	407
11.6.4	Frequency Response Functions	408
11.6.5	Unit Impulse Response (Weighting) Functions	409
11.6.6	Ordinary Coherence Functions	409
11.7	MULTIPLE-INPUT / OUTPUT FUNCTIONS	409
11.7.1	Fourier Transforms and Spectral Functions	411
11.7.2	Conditioned Spectral Density Functions	411
11.7.3	Frequency Response and Coherence Functions	415
11.7.4	Coherent Output Spectral Functions	417
11.7.5	Decomposition of Measured Spectral Functions	418
12	NONSTATIONARY DATA ANALYSIS	425
12.1	CLASSES OF NONSTATIONARY DATA	425
12.2	PROBABILITY STRUCTURE OF NONSTATIONARY DATA	428
12.2.1	Higher-Order Probability Functions	429
12.2.2	Time-Averaged Probability Functions	430
12.3	NONSTATIONARY MEAN VALUES	432
12.3.1	Independent Samples	433
12.3.2	Correlated Samples	434
12.3.3	Analysis Procedures for Single Records	436
12.4	NONSTATIONARY MEAN SQUARE VALUES	438
12.4.1	Independent Samples	438
12.4.2	Correlated Samples	440
12.4.3	Analysis Procedures for Single Records	441
12.5	CORRELATION STRUCTURE OF NONSTATIONARY DATA	443
12.5.1	Double Time Correlation Functions	443

12.5.2	Alternative Double Time Correlation Functions	444
12.6	SPECTRAL STRUCTURE OF NONSTATIONARY DATA	447
12.6.1	Double Frequency Spectral Functions	448
12.6.2	Alternative Double Frequency Spectral Functions	450
12.6.3	Frequency–Time Spectral Functions	456
12.6.4	Product Model Nonstationary Data	465
12.7	INPUT / OUTPUT RELATIONS FOR NONSTATIONARY DATA	471
12.7.1	Nonstationary Input and Time-Varying System	472
12.7.2	Results for Special Cases	474
12.7.3	Frequency–Time Input/Output Relations	475
12.7.4	Energy Spectral Input/Output Relations	477
13	THE HILBERT TRANSFORM	484
13.1	HILBERT TRANSFORMS FOR GENERAL RECORDS	484
13.1.1	Computation of Hilbert Transforms	487
13.1.2	Examples of Hilbert Transforms	488
13.1.3	Properties of Hilbert Transforms	489
13.1.4	Relation to Physically Realizable Systems	492
13.2	HILBERT TRANSFORMS FOR CORRELATION FUNCTIONS	496
13.2.1	Correlation and Envelope Definitions	496
13.2.2	Hilbert Transform Relations	498
13.2.3	Analytic Signals for Correlation Functions	499
13.2.4	Nondispersive Propagation Problems	502
13.2.5	Dispersive Propagation Problems	506
13.3	ENVELOPE DETECTION FOLLOWED BY CORRELATION	510
	REFERENCES	517
	APPENDIX A: STATISTICAL TABLES	521
	APPENDIX B: DEFINITIONS FOR RANDOM DATA ANALYSIS	534
	LIST OF FIGURES	547
	LIST OF TABLES	553
	LIST OF EXAMPLES	555
	INDEX	559