

Contents

Acknowledgements	xi
Preface	xiii
Part 1 Introduction	1
Chapter 1 Hypercomplex Iterations in a Nutshell	3
Chapter 2 Deterministic Fractals and Distance Estimation	11
2.1. Fractals and Visualization	11
2.2. Deterministic Fractals, Julia Sets and Mandelbrot Sets	12
2.3. Distance Estimation	13
Part 2 Classical Analysis: Complex and Quaternionic	15
Chapter 3 Distance Estimation in Complex Space	17
3.1. Complex Dynamical Systems	17
3.2. The Quadratic Family, Julia Sets and the Mandelbrot Set	18
3.3. The Distance Estimation Formula	21

3.4.	Schwarz's Lemma and an Upper Bound of the Distance Estimate	23
3.5.	The Koebe 1/4 Theorem and a Lower Bound for the Distance Estimate	29
3.6.	An Approximation of the Distance Estimation Formula	35
Chapter 4 Quaternion Analysis		39
4.1.	The Quaternions	40
4.2.	Rotations of 3-Space	41
4.3.	Quaternion Polynomials	42
4.4.	Quaternion Julia Sets and Mandelbrot Sets ⁻	43
4.5.	Differential Forms	44
4.6.	Regular Functions	47
4.7.	Cauchy's Theorem and the Integral Formula	48
4.8.	Linear and Quadratic Regular Functions	53
4.9.	Difficulties of the Quaternion Analytic Proof of Distance Estimation	54
Chapter 5 Quaternions and the Dirac String Trick		57
Part 3 Hypercomplex Iterations		63
Chapter 6 Quaternion Mandelbrot Sets		65
6.1.	Quaternion Mandelbrot Sets	65
6.2.	The Distance Estimate for Quaternion Mandelbrot Sets	65
Chapter 7 Distance Estimation in Higher Dimensional Spaces		71
7.1.	Higher Dimensional Deterministic Fractals	71
7.2.	The Cayley Numbers	73
7.3.	Distance Estimation in Higher Dimensional Spaces	74
7.4.	Calculating the Derivative in Higher Dimensional Space	77
7.5.	Another Version of the Distance Estimation Formula	82

Part 4	Inverse Iteration, Ray Tracing and Virtual Reality	89
Chapter 8	Inverse Iteration: An Interactive Visualization	91
8.1.	Classical Inverse Iteration	91
8.2.	Mappings in the Quaternions	93
8.3.	The Quaternion Square Root	94
8.4.	The n th Roots in Higher Dimensions	96
8.5.	Quaternion Julia Sets via Inverse Iteration	97
8.6.	Functions Used in the Inverse Iteration Method	98
8.7.	An Algorithm for the Inverse Iteration Method	100
8.8.	Tree Pruning	102
8.9.	Displaying Julia Sets	104
Chapter 9	Ray Tracing Methods by Distance Estimation	107
9.1.	Distance Estimation via Ray Tracing	107
9.2.	A Classical Ray Tracing Algorithm	108
9.3.	A Ray Tracing Algorithm Using Distance Estimation	108
9.4.	Quaternion Multiplication in the Algorithm	110
9.5.	Calculating the Derivative in the Algorithm	111
9.6.	Some Important Parameters in the Algorithm	113
9.7.	The n th power Family of Quaternion Mandelbrot Sets	114
9.8.	The Quadratic Family of Julia Sets	116
9.9.	Generalized Quaternion Julia Sets	118
9.10.	Disconnected Quaternion Julia Sets	121
9.11.	Displaying and Rendering	122
9.11.1.	Light models	122
9.11.2.	Surface normal	123
9.11.3.	Clarity	125
9.11.4.	Other Rendering Considerations	127

Chapter 10	Quaternion Deterministic Fractals in Virtual Reality	129
10.1.	Introduction to Virtual Reality	129
10.2.	Parallel Computation	130
10.3.	Data Communication	131
10.4.	An Improved Display Algorithm	132
10.5.	Display of Quaternion Deterministic Fractals in VR . . .	133
10.6.	Conclusion	134
	Appendix A	135
	Appendix B	137
	Bibliography	139
	Index	143