Contents

Notation and Symbols xi

1. Introduction 1

2. Optimization of a Simple Servomechanism 5

2.1 Description of the Simple Servomechanism 5
2.2 The Performance Criterion 7
2.3 Adjustable Damping 9
2.4 Adjustable Gain 10
2.5 Two Adjustable Parameters 11
2.6 The Method of Lagrange Multipliers 12
2.7 Graphical Interpretation of the Minimization Process 13

3. Statistical Design of Linear Systems 17

3.1 Statistical Description of Random Signals 17
3.2 Mean Squared Error in Terms of Correlation Functions 21
3.3 Frequency-Domain Characteristics of Random Signals 22
3.4 Frequency-Domain Computation of Mean Squared Values of Random Functions 24
3.5 Optimization of the Parameters of a System 26
3.6 Use of Transient Signals To Find the Mean Squared Error for a Random Input 29
3.7 Optimization of Impulse Response 31
3.8 Solution of the Wiener-Hopf Equation 36
3.9 A Simple Example of Optimal Filtering 41
3.10 Optimal Filtering of Nonstationary Processes 43
3.11 The Kalman Filter 47

4. State Representation of Systems 58

4.1 Representation of System by State Equations 58
4.2 Systems with Transfer-Function Zeros 62

	4.3	Eigenvalues and Eigenvectors of a Matrix	63
	4.4	Response of a System in Terms of Eigenvalues and Eigenvectors	68
	4.5	Response of a System in Terms of Functions of a Matrix	73
	4.6	Fundamental Matrices	75
	4.7	Methods of Evaluating the Transition Matrix	76
	4.8	Response of a Linear System with Nonzero Input	83
	4.9	Transfer-Function Matrices	85
	4.10	Controllability and Observability	86
	4.11	Analogue-Computer Simulation of Systems	92
	4.12	State Representation of Linear Sampled-Data Systems	94
	4.13	Linear Time-Varying Systems	99
	4.14	Nonlinear Systems	101
5.	**Calculus of Variations**		**110**
	5.1	Maximization or Minimization of a Functional	111
	5.2	Variable Endpoints	115
	5.3	A Problem in Optimal Control	119
	5.4	Problems with Isoperimetric Constraints	121
	5.5	Problems with More Than One Dependent Variable	124
	5.6	The Mayer Problem	128
	5.7	A High-Order Control Problem	131
	5.8	The Hamiltonian Formulation	132
6.	**The Maximum Principle**		**138**
	6.1	Constraints on the Control Inputs	138
	6.2	The Maximum Principle	140
	6.3	The Minimum-Time Control Problem	143
	6.4	General Considerations in Minimum-Time Control of Linear Systems	154
	6.5	Geometrical Interpretation of the Transversality Condition	156
	6.6	Further Examples Illustrating the Transversality Condition	159
	6.7	Minimum-Fuel Control	164
7.	**Dynamic Programming**		**171**
	7.1	Multistage Decision Processes	171
	7.2	Use of the Principle of Optimality	172

7.3 A Discrete-Time System 175
7.4 Continuous Version of Dynamic Programming 187
7.5 A Continuous Control Problem 190
7.6 Dynamic Programming and the Maximum Principle 196

8. Computational Methods of Optimization 204
8.1 Derivation of an Optimum Search Procedure 204
8.2 Newton-Raphson Method of Maximization 212
8.3 Methods Which Do Not Involve Derivatives 213
8.4 Extension to More Than One Independent Variable 214
8.5 Hill-Climbing Methods of Solution of Equations 217
8.6 Computational Optimization Using the Maximum Principle 218
8.7 Direct Computation of an Optimal Control Input 220
8.8 Analogue-Computer Solution of an Optimal Control Problem 223

Bibliography 229

Appendices 233
A. Parseval's Theorem 233
B. Some Remarks on the Bilateral Laplace Transformation 234
C. Some Relationships Involving Autocorrelation Functions and Spectral Densities 238
D. Table of Integrals 240
E. Answers to Problems 242

Index 255