

## CONTENTS

PREFACE . . . . .		v
CHAPTER 1. INTRODUCTION . . . . .		1
1.1 General Nature of the Optimum Design Problem . . . . .		1
1.2 The Role of Statistical Methods of Analysis . . . . .		3
1.3 Stationary and Nonstationary Problems. . . . .		4
1.4 Performance Index of a System . . . . .		5
1.5 Organization of Text . . . . .		7
CHAPTER 2. BASIC CONCEPTS OF PROBABILITY THEORY . . . . .		9
2.1 Introduction. . . . .		9
2.2 Sets and Set Operations . . . . .		10
2.3 Point Functions and Set Functions . . . . .		14
2.4 Axioms of Probability . . . . .		17
2.5 Independence of Events and Conditional Probability . . . . .		22
2.6 Random Variables . . . . .		28
2.7 Probability Distribution and Frequency Functions. . . . .		34
2.8 Expectation, Mean, Variance, and Moments . . . . .		45
2.9 Characteristic Functions . . . . .		57
2.10 The Binomial Distribution . . . . .		62
2.11 The Poisson Distribution . . . . .		63
2.12 The Normal Distribution and the Central-limit Theorem . . . . .		67
2.13 The Multidimensional Normal Distribution. . . . .		73
CHAPTER 3. THE STATISTICAL DESCRIPTION OF RANDOM PROCESSES . . . . .		90
3.1 Introductory Remarks . . . . .		90
3.2 Random Processes . . . . .		92
3.3 Probability Distributions and Statistical Parameters . . . . .		94
3.4 Joint Distributions for Two or More Random Processes . . . . .		100
3.5 Stationary and Ergodic Random Processes . . . . .		105
3.6 Power Spectral Density . . . . .		120
3.7 Further Examples of Random Processes. . . . .		136
CHAPTER 4. THE SHOT EFFECT AND GAUSSIAN RANDOM PROCESSES. . . . .		147
4.1 The Shot Effect . . . . .		147
4.2 The Gaussian Random Process . . . . .		155
4.3 The Empirical Determination of Correlation Functions . . . . .		160
4.4 Correlation Function of the Output of Simple Nonlinear Devices . . . . .		163
4.5 Analysis of an Automatic Tracking System . . . . .		171

CHAPTER 5.	ANALYSIS OF EFFECTS OF TIME-INVARIANT LINEAR SYSTEMS ON STATIONARY RANDOM PROCESSES . . . . .	177
5.1	Filtering and Prediction . . . . .	177
5.2	The Response Characteristics of Linear Systems . . . . .	182
5.3	Input-Output Relations for Correlation Functions and Spectral Densities . . . . .	194
5.4	A General Class of Filtering and Prediction Problems . . . . .	201
5.5	The Analytical Computation of the Mean-squared Error . . . . .	207
5.6	Analogue Computation Techniques . . . . .	218
CHAPTER 6.	MEAN-SQUARED ERROR ANALYSIS FOR NONSTATIONARY PROBLEMS . . . . .	225
6.1	General Integral Formulas for Mean-squared Errors . . . . .	225
6.2	Transient Statistical Analysis of a Time-invariant Linear System . . . . .	230
6.3	Frequency Methods for Transient Analysis of Linear Systems . . . . .	234
6.4	The Method of Adjoint Systems . . . . .	239
6.5	Continuous Generation of Mean-squared Errors . . . . .	247
6.6	Optimum Design Procedure for a Class of Variable-coefficient Systems by a Method of Steepest Descents. . . . .	253
CHAPTER 7.	OPTIMUM LINEAR LEAST-SQUARES SMOOTHING AND PREDICTION FOR STATIONARY RANDOM PROCESSES . . . . .	269
7.1	Formulation of the Wiener Smoothing and Prediction Problem . . . . .	269
7.2	Pure Prediction . . . . .	272
7.3	The Method of Bode and Shannon . . . . .	276
7.4	Analytic Solution of the Wiener-Hopf Equation Using Complex Variable Techniques. . . . .	280
7.5	Direct Solution of the Wiener-Hopf Equation for Rational Spectra . . . . .	283
CHAPTER 8.	OPTIMUM OPERATIONS WITH FINITE DATA. . . . .	291
8.1	Introductory Remarks . . . . .	291
8.2	Prediction Involving Unknown Linear Combinations of Known Functions in the Presence of Noise . . . . .	300
8.3	Prediction Involving Statistically Known Linear Combinations of Known Functions in the Presence of Noise . . . . .	305
8.4	The Integral Equation for the Stationary Case. . . . .	309
8.5	The Integral Equation for the Nonstationary Case . . . . .	329
8.6	Prediction and Filtering in the Presence of Gaussian Interference . . . . .	343
APPENDIX A.	Evaluation of an Integral Occurring in the Analysis of Certain Nonlinear Devices . . . . .	359
APPENDIX B.	Stationary Gaussian Noise through a Limiter. . . . .	361
APPENDIX C.	Analogue Computers . . . . .	366
C.1	Introduction . . . . .	366
C.2	Theory of Operation of the Linear Computing Elements . . . . .	366

C.3	Theory of Operation of the Nonlinear Computing Elements . . . . .	369
C.4	Scale-factor Techniques . . . . .	372
C.5	A Useful Computing Technique . . . . .	376
APPENDIX D.	A Class of Orthogonal Functions . . . . .	381
APPENDIX E.	Evaluation of an Integral Occurring in Mean-squared Error Analysis of Linear Systems . . . . .	395
APPENDIX F.	Mathematical Proof of the Method of Adjoint Systems . . . . .	398
APPENDIX G.	Derivation of Optimum Time-varying Parameter for First-order Problem . . . . .	404
APPENDIX H.	Solution of the Equation $P(x) = 0$ . . . . .	412
APPENDIX I.	Delta-function Terms in the Solution of a Certain Integral Equation . . . . .	416
APPENDIX J.	Inversion of a Certain Matrix . . . . .	419
BIBLIOGRAPHY	. . . . .	424
INDEX	. . . . .	431