

# Contents

|   |    |
|---|----|
| 1. INTRODUCTION                                       |    |
| Numerical analysis                                    | 1  |
| Computer arithmetic                                   | 5  |
| Simple error analysis                                 | 7  |
| Computing machines, programming and coding            | 9  |
| Checking  | 15 |
| Additional notes                                      | 16 |
| 2. MATRIX ALGEBRA                                     |    |
| Introduction  | 18 |
| Linear equations. General considerations              | 18 |
| Homogeneous equations                                 | 20 |
| Linear equations and matrices                         | 21 |
| Matrix addition and multiplication                    | 22 |
| Inversion and solution. The unit matrix               | 26 |
| Transposition and symmetry. Inversion of products     | 29 |
| Some special matrices                                 | 31 |
| Triangular matrices. The decomposition theorem        | 32 |
| The determinant                                       | 34 |
| Cofactors and the inverse matrix                      | 36 |
| Determinants of special matrices                      | 37 |
| Partitioned matrices                                  | 38 |
| Latent roots and vectors                              | 39 |
| Similarity transformations                            | 42 |
| Orthogonality   | 44 |
| Symmetry, Rayleigh's Principle. Hermitian matrices    | 45 |
| Limits, series and norms                              | 48 |
| Numerical methods                                     | 51 |
| Additional notes and bibliography                     | 55 |
| 3. ELIMINATION METHODS OF GAUSS, JORDAN<br>AND AITKEN |    |
| Introduction  | 60 |
| The Gauss methods                                     | 61 |
| Jordan elimination                                    | 65 |
| Calculation of the inverse                            | 66 |
| Matrix equivalent of elimination                      | 68 |
| The method of Aitken                                  | 75 |
| The symmetric case                                    | 79 |
| The symmetric, positive-definite case                 | 80 |
| Exact and approximate solutions. Integer coefficients | 82 |
| Determination of rank                                 | 87 |
| Complete pivoting                                     | 91 |
| Compatibility of linear equations                     | 93 |
| Note on comparison of methods                         | 96 |
| Additional notes and bibliography                     | 97 |

|   |     |
|---|-----|
| 4. COMPACT ELIMINATION METHODS OF DOOLITTLE, CROUT, BANACHIEWICZ AND CHOLESKY |     |
| Introduction  | 99  |
| The method of Doolittle   | 99  |
| Connexion with decomposition  | 102 |
| The method of Crout   | 102 |
| Symmetric case  | 104 |
| The methods of Banachiewicz and Cholesky                                      | 106 |
| Inversion. Connexion with Doolittle and Crout                                 | 110 |
| Inversion. Symmetric case   | 113 |
| Connexion with Jordan and Aitken  | 115 |
| Row interchanges  | 117 |
| Operations with complex matrices  | 121 |
| Additional notes and bibliography   | 124 |
| 5. ORTHOGONALISATION METHODS  |     |
| Introduction  | 125 |
| Symmetric case  | 126 |
| Unsymmetric case  | 128 |
| Matrix orthogonalisation  | 130 |
| Additional notes and bibliography   | 135 |
| 6. CONDITION, ACCURACY AND PRECISION  |     |
| Introduction  | 136 |
| Symptoms, causes and effects of ill-conditioning                              | 137 |
| Measure of condition  | 141 |
| Exact and approximate data  | 143 |
| Mathematical problems. Correction to approximate solution                     | 143 |
| Mathematical problems. Correction to the inverse                              | 155 |
| Physical problems. Error analysis   | 158 |
| Relative precision of components of solution                                  | 167 |
| Additional notes and bibliography   | 169 |
| 7. COMPARISON OF METHODS. MEASURE OF WORK                                     |     |
| Introduction  | 175 |
| Gauss elimination   | 175 |
| Jordan elimination  | 179 |
| Matrix decomposition  | 180 |
| Aitken elimination  | 183 |
| Other elimination methods. Symmetry   | 183 |
| Evaluation and comparison   | 185 |
| Additional notes  | 186 |
| 8. ITERATIVE AND GRADIENT METHODS   |     |
| Introduction  | 189 |
| General nature of iteration   | 190 |
| Jacobi and Gauss-Seidel iteration   | 191 |
| Acceleration of convergence   | 194 |
| Labour and accuracy   | 202 |
| Consistent ordering   | 203 |
| Gradient methods  | 205 |

|   |            |
|---|------------|
| Symmetric positive-definite case                                | 207        |
| A finite iterative process                                      | 208        |
| Additional notes and bibliography                               | 213        |
| <b>9. ITERATIVE METHODS FOR LATENT ROOTS AND VECTORS</b>        |            |
| Introduction  | 215        |
| Direct iteration  | 216        |
| Acceleration of convergence                                     | 220        |
| Other roots and vectors. Inverse iteration                      | 223        |
| Matrix deflation  | 228        |
| Connexion with similarity transformation                        | 232        |
| Additional notes and bibliography                               | 233        |
| <b>10. TRANSFORMATION METHODS FOR LATENT ROOTS AND VECTORS</b>  |            |
| Introduction  | 238        |
| Method of Jacobi, symmetric matrices                            | 238        |
| Method of Givens, symmetric matrices                            | 241        |
| Method of Householder, symmetric matrices                       | 247        |
| Example of Givens and Householder                               | 249        |
| Uniqueness of triple-diagonal form                              | 251        |
| Method of Lanczos, symmetric matrices                           | 252        |
| Method of Lanczos, unsymmetric matrices                         | 255        |
| Vectors of triple-diagonal matrices                             | 258        |
| Other similarity transformations. The L-R method                | 259        |
| The Q-R method  | 263        |
| Reduction to Hessenberg form                                    | 264        |
| Roots and vectors of Hessenberg matrix                          | 266        |
| Additional notes and bibliography                               | 268        |
| <b>11. NOTES ON ERROR ANALYSIS FOR LATENT ROOTS AND VECTORS</b> |            |
| Introduction  | 275        |
| Ill-conditioning  | 275        |
| Corrections to approximate roots and vectors                    | 278        |
| General perturbation analysis                                   | 281        |
| Deflation perturbation  | 286        |
| Additional notes and bibliography                               | 288        |
| <b>INDEX</b>  | <b>291</b> |

NOTE. † indicates that there is a further mention of the section in *Additional notes and bibliography*, given at the end of each Chapter.