

Contents

Foreword	xiii
Preface	xv
Notation	xix

1. The Matrix Eigenvalue Problem

Introduction	1
1. The Eigenvalue Problem in \mathbb{C}^N	1
2. The Stability of the Eigenvalue Problem	11
3. Some Numerical Methods	19
4. Error Analysis	30
5. Large-Eigenvalue Problems	48

2. Elements of Functional Analysis: Basic Concepts

Introduction	66
<i>A. Bounded and Closed Operators</i>	
1. Banach and Hilbert Spaces	66
2. Adjoint Space	68
3. Compact Sets in a Banach Space	74
4. Bounded Linear Operators	75
5. Pairs of Projections and the Gap between Subspaces	86
6. Closed Linear Operators	89
<i>B. Introduction to Spectral Theory</i>	
7. Resolvent and Spectrum	95
8. Operators in a Hilbert Space	114
9. Spectrum of Compact Operators and Operators with Compact Resolvent	117

3. Elements of Functional Analysis: Convergence and Perturbation Theory

Introduction	121
<i>A. Convergence of a Sequence of Operators</i>	
1. Convergence of a Sequence of Operators in $\mathcal{L}(X)$	122

2. Properties of the Convergences in $\mathcal{L}(X)$	123
3. Summary	129
4. Convergence of a Sequence of Operators in $\mathcal{C}(X)$	130
5. Properties of the Convergences in $\mathcal{C}(X)$	132
6. Summary	136
<i>B. Analytic Perturbation Theory</i>	
7. Analyticity of $R(t, z)$, $P(t)$, and $\hat{\lambda}(t)$	140
8. Iterative Computation of the Coefficients of the Series Expansions	143
4. Numerical Approximation Methods for Integral and Differential Operators	
Introduction	163
<i>A. Fredholm Integral Operators</i>	
1. The Problem	164
2. Projections and Numerical Quadratures	166
3. Projection Methods	170
4. Approximate Quadrature Methods	178
5. The Iterated Solution and Eigenvector	181
6. Abstract Setting for the Approximation Operators	183
7. Convergence of the Numerical Approximation Methods	189
<i>B. Differential Boundary-Value Problems</i>	
8. Boundary-Value Problems in Differential Equations	199
9. Projection Methods for an Ordinary Differential Equation	206
10. The Projection Method for Partial Differential Equations	213
11. Finite-Difference Methods	222
12. Approximation of a Differential Operator by a Neighboring Operator	226
5. Spectral Approximation of a Closed Linear Operator	
Introduction	228
1. Convergence of the Spectrum $\sigma(T_n) \cap \Delta$	229
2. Convergence of the Eigenvalues with Preservation of the Multiplicities	234
3. Convergence of the Eigenvectors and Invariant Subspaces	235
4. T and T_n Are Self-Adjoint in a Hilbert Space H	239
5. Strong Stability of an Approximation of a Closed Operator	243
6. Iterative Refinement Method When $T_n - z \xrightarrow{uo} T - z$ in $\rho(T)$	253
6. Error Bounds and Localization Results for the Eigelements	
Introduction	277
1. Theoretical Error Bounds	278
2. The Projection Method	283
3. One Example: The Finite-Element Method	288
4. A Posteriori Error Bounds for Bounded Operators	293

5. Localization of a Group of Eigenvalues of T	302
6. Asymptotic Behavior of the Constants in the Error Bounds	314

7. Some Examples of Applications

Introduction	320
<i>A. Superconvergence Results for Integral and Differential Equations</i>	
1. Definition of the Problem	323
2. Smoothness Properties of the Solutions	325
3. Superconvergence Results for the \perp -Galerkin Method	330
4. Connection between the \perp -Galerkin and Collocation Methods	333
5. Superconvergence Results for the Collocation Method at Gauss Points	335
6. Superconvergence at the Partition Points of the Approximate Solution of an Ordinary Differential Equation	342
7. Superconvergence for the Differential Eigenvalue Problem	346
<i>B. Iterative Refinement for the Eigenelements</i>	
8. T Is an Integral Operator	353
9. T Is a Differential Operator	361

Appendix. Discrete Approximation Theory

1. Discrete Approximation of a Banach Space	365
2. Discrete Approximation of a Closed Operator	368

References 369

Solutions to Exercises 404

By M. Ahués

Notation Index	451
Subject Index	453