
Contents

Preface	ix
Part 1. Classical theory	
Chapter 1. Introduction	3
§1.1. Newton's equations	3
§1.2. Classification of differential equations	6
§1.3. First-order autonomous equations	9
§1.4. Finding explicit solutions	13
§1.5. Qualitative analysis of first-order equations	20
§1.6. Qualitative analysis of first-order periodic equations	28
Chapter 2. Initial value problems	33
§2.1. Fixed point theorems	33
§2.2. The basic existence and uniqueness result	36
§2.3. Some extensions	39
§2.4. Dependence on the initial condition	42
§2.5. Regular perturbation theory	48
§2.6. Extensibility of solutions	50
§2.7. Euler's method and the Peano theorem	54
Chapter 3. Linear equations	59
§3.1. The matrix exponential	59
§3.2. Linear autonomous first-order systems	66
§3.3. Linear autonomous equations of order n	74

§3.4.	General linear first-order systems	80
§3.5.	Linear equations of order n	87
§3.6.	Periodic linear systems	91
§3.7.	Perturbed linear first-order systems	97
§3.8.	Appendix: Jordan canonical form	103
Chapter 4.	Differential equations in the complex domain	111
§4.1.	The basic existence and uniqueness result	111
§4.2.	The Frobenius method for second-order equations	116
§4.3.	Linear systems with singularities	130
§4.4.	The Frobenius method	134
Chapter 5.	Boundary value problems	141
§5.1.	Introduction	141
§5.2.	Compact symmetric operators	146
§5.3.	Sturm–Liouville equations	153
§5.4.	Regular Sturm–Liouville problems	155
§5.5.	Oscillation theory	166
§5.6.	Periodic Sturm–Liouville equations	175
Part 2. Dynamical systems		
Chapter 6.	Dynamical systems	187
§6.1.	Dynamical systems	187
§6.2.	The flow of an autonomous equation	188
§6.3.	Orbits and invariant sets	192
§6.4.	The Poincaré map	197
§6.5.	Stability of fixed points	198
§6.6.	Stability via Liapunov’s method	201
§6.7.	Newton’s equation in one dimension	203
Chapter 7.	Planar dynamical systems	209
§7.1.	Examples from ecology	209
§7.2.	Examples from electrical engineering	215
§7.3.	The Poincaré–Bendixson theorem	220
Chapter 8.	Higher dimensional dynamical systems	229
§8.1.	Attracting sets	229
§8.2.	The Lorenz equation	234

§8.3. Hamiltonian mechanics	238
§8.4. Completely integrable Hamiltonian systems	243
§8.5. The Kepler problem	247
§8.6. The KAM theorem	250
Chapter 9. Local behavior near fixed points	255
§9.1. Stability of linear systems	255
§9.2. Stable and unstable manifolds	257
§9.3. The Hartman–Grobman theorem	264
§9.4. Appendix: Integral equations	270
Part 3. Chaos	
Chapter 10. Discrete dynamical systems	281
§10.1. The logistic equation	281
§10.2. Fixed and periodic points	284
§10.3. Linear difference equations	287
§10.4. Local behavior near fixed points	288
Chapter 11. Discrete dynamical systems in one dimension	293
§11.1. Period doubling	293
§11.2. Sarkovskii’s theorem	296
§11.3. On the definition of chaos	297
§11.4. Cantor sets and the tent map	300
§11.5. Symbolic dynamics	303
§11.6. Strange attractors/repellers and fractal sets	309
§11.7. Homoclinic orbits as source for chaos	313
Chapter 12. Periodic solutions	317
§12.1. Stability of periodic solutions	317
§12.2. The Poincaré map	319
§12.3. Stable and unstable manifolds	321
§12.4. Melnikov’s method for autonomous perturbations	324
§12.5. Melnikov’s method for nonautonomous perturbations	329
Chapter 13. Chaos in higher dimensional systems	333
§13.1. The Smale horseshoe	333
§13.2. The Smale–Birkhoff homoclinic theorem	335
§13.3. Melnikov’s method for homoclinic orbits	336

Bibliographical notes	341
Bibliography	345
Glossary of notation	349
Index	351