

Contents

Chapter 1. Introduction	1
Chapter 2. Statement of the results	3
2.1. Random graph estimates	4
Chapter 3. Mixing time preliminaries	7
Chapter 4. Outline of the proof of Theorem 2.1	9
4.1. Outline of the Proof of Theorem 2.1 (i)	9
4.2. Outline of the Proof of Theorem 2.1 (iii)	10
4.3. Outline of the Proof of Theorem 2.1 (ii)	10
Chapter 5. Random graph estimates	13
5.1. The exploration process	13
5.2. Random graph lemmas for non-critical cases	14
5.3. Random graph lemmas for the near-critical case	19
Chapter 6. Supercritical case	37
Chapter 7. Subcritical case	45
Chapter 8. Critical Case	49
8.1. Starting at the $[n^{3/4}, n]$ regime: Proof of Theorem 8.1	50
8.2. Starting at the $[0, n^{3/4}]$ regime: Proof of Theorem 8.2	71
8.3. The lower bound on the mixing time	77
Chapter 9. Fast mixing of the Swendsen-Wang process on trees	79
Acknowledgements	81
Bibliography	83