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Frontispiece. Title-page of THE ELEMENTS of *Geometrie of the most auncient Philosopher EVCLIDE of Megara* by H. Billingsley (London 1570)
(From the copy in the University Library, Cambridge)

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