

Contents

On the Positive Mass, Penrose, and ZAS Inequalities in General

Dimension

<i>Hubert L. Bray</i>	1
1 Dedication	1
2 Introduction	2
3 A Trio of Inequalities	15
References	25

Recent Progress on the Yamabe Problem

<i>Simon Brendle, Fernando C. Marques</i>	29
1 The Yamabe Problem	29
2 The Compactness Conjecture	31
3 Non-compactness Results in Dimension $n \geq 25$	34
4 A Compactness Result in Dimension $n \leq 24$	37
5 The Parabolic Yamabe Flow	40
References	45

Some Recent Progress on Mean Curvature Flow for Entire Lagrangian Graphs

<i>Jingyi Chen</i>	49
1 Introduction	49
2 Longtime Existence With Lipschitz Continuous Initial Data	50
3 Uniqueness and Viscosity Solutions	52
4 Self-similar Solutions	53
References	57

Radial Viewpoint on Minimal Surfaces

<i>Jaigyoung Choe</i>	59
1 Introduction	59
2 Cone	60
3 Horizon	62
4 Non-Euclidean Space	63
5 Ray preserving Metric	65
6 Varying Curvature	67
7 Embeddedness	70

References.....	72
Minimal Surfaces and Mean Curvature Flow	
<i>Tobias H. Colding, William P. Minicozzi II</i>	73
1 Introduction.....	73
2 Harmonic Functions and the Heat Equation.....	74
3 Energy of a Curve.....	78
4 Birkhoff: A Closed Geodesic on a Two Sphere.....	80
5 Curve Shortening Flow.....	84
6 Minimal Surfaces.....	88
7 Classification of Embedded Minimal Surfaces.....	96
8 Mean Curvature Flow.....	112
9 Width and mean curvature flow.....	117
10 Singularities for MCF.....	119
11 Smooth Compactness Theorem for Self-shrinkers.....	124
12 The Entropy.....	126
13 An Application.....	131
14 Non-compact self-shrinkers.....	132
References.....	135
Scalar Curvature and the Einstein Constraint Equations	
<i>Justin Corvino, Daniel Pollack</i>	145
1 Introduction.....	145
2 The Constraint Equations.....	147
3 A Tour of Asymptotically Flat Solutions.....	149
4 The Conformal Method.....	167
5 Gluing Constructions.....	172
References.....	182
On the Intrinsic Differentiability Theorem of Gromov-Schoen	
<i>Georgios Daskalopoulos, Chikako Mese</i>	189
1 Introduction.....	189
2 Definitions.....	190
3 Main Theorem.....	192
References.....	203
Minimal Surface Techniques in Riemannian Geometry	
<i>Ailana Fraser</i>	205
1 Introduction.....	205
2 Brief Overview of Some Geodesic Methods.....	206
3 Existence of Minimal Surfaces.....	208
4 Second Variation Theory for Minimal Surfaces and Applications..	211

References.....	217
Stability and Rigidity of Extremal Surfaces in Riemannian Geometry and General Relativity	
<i>Gregory J. Galloway</i>	221
1 Minimal Hypersurfaces in Manifolds of Nonnegative Scalar Curvature.....	221
2 Marginally Outer Trapped Surfaces.....	226
3 Positivity of Mass for Asymptotically Hyperbolic Manifolds.....	232
References.....	237
Convex Hypersurfaces of Constant Curvature in Hyperbolic Space	
<i>Bo Guan, Joel Spruck</i>	241
1 Introduction.....	241
2 Formulas on Hypersurfaces.....	246
3 The Asymptotic Angle Maximum Principle and Gradient Estimates.....	250
4 Curvature Estimates.....	251
5 Uniqueness and Foliations.....	254
References.....	257
Ricci Flow in Two Dimensions	
<i>James Isenberg, Rafe Mazzeo, Natasa Sesum</i>	259
1 Introduction.....	259
2 General Considerations.....	261
3 Compact Surfaces.....	262
4 Open Surfaces.....	267
5 Flows on Incomplete Surfaces.....	274
References.....	278
Doubling and Desingularization Constructions for Minimal Surfaces	
<i>Nikolaos Kapouleas</i>	281
1 Introduction.....	281
2 Doubling Constructions.....	288
3 Desingularization Constructions.....	296
4 Minimal Surfaces in the Round Three-Sphere.....	299
5 The Building Blocks for the Desingularization Construction.....	304
6 An Initial Surface for the Desingularization Construction.....	309
7 The Family of Initial Surfaces for the Desingularization Construction.....	312
8 Main Estimates and Outline of the Proof.....	317
References.....	322

The Metric Properties of Lagrangians	
<i>Yng-Ing Lee</i>	327
1 Introduction	327
2 A Short Survey	328
3 Definitions and Properties	332
4 Singularities and Geometric Measure Theory	334
5 Gluing and Singular Perturbation	336
References	338
Structure of Complete Manifolds with Positive Spectrum	
<i>Peter Li</i>	343
1 Introduction	343
2 Riemannian Case	344
3 Kähler Case	348
4 Quaternionic Kähler Manifolds, Cayley Manifolds, and Locally Symmetric Spaces	351
5 Manifolds of Finite Volume	354
6 Further Generalizations	356
References	360
Topology of Sobolev Mappings and Associated Variational Problems	
<i>Fang Hua Lin</i>	363
Introduction	363
1 Analytical and Topological Properties of Sobolev Maps	364
2 Singularity of Energy Minimizing Maps	373
3 Limits of Singular Sets of p -Energy Minimizing Maps	380
References	391
A Survey of Research on Boundary Behavior of Compact Manifolds via the Positive Mass Theorem	
<i>Pengzi Miao</i>	395
1 Introduction	395
2 Statement of the Positive Mass Theorem	395
3 On compact Manifolds with Nonnegative Scalar Curvature	397
4 On Compact Manifolds with Negative Scalar Curvature	406
References	410
Recent Progress on Singularities of Lagrangian Mean Curvature Flow	
<i>André Neves</i>	413
1 Introduction	413
2 Preliminaries	415

3	Basic Techniques.....	416
4	Applications I: Blow-ups.....	424
5	Applications II: Self-Expanders.....	428
6	Application III: Stability of Singularities.....	430
7	Open Questions.....	434
	References.....	436
Geometric Structures of Collapsing Riemannian Manifolds I		
	<i>Aaron Naber, Gang Tian</i>	439
1	Introduction.....	439
2	Structure of Collapsed Spaces.....	443
3	Geometry of Toric Quotients.....	448
4	Geometry of Toric Quotients II.....	454
5	Proof of Theorems 1.1 and 1.2.....	458
6	Proof of Theorem 1.3.....	457
A	Geometry of Quotients.....	457
B	Orbifolds.....	460
	References.....	465
Deformation of Kähler-Einstein Metrics		
	<i>Xiaofeng Sun, Shing-Tung Yau</i>	467
1	Introduction.....	467
2	Complex Structures of Kähler-Einstein Manifolds.....	468
3	Deformation of Kähler-Einstein Metrics.....	473
4	Local Trivialization of Polarization Bundles and Deformation of Sections.....	476
5	Curvature of L^2 Metrics on Direct Image Sheaves.....	483
6	Appendix.....	486
	References.....	489
Reverse Bubbling in Geometric Flows		
	<i>Peter M. Topping</i>	491
1	Introduction.....	491
2	The Harmonic map Flow.....	495
3	Ricci Flow.....	499
4	Addendum — Mean Curvature Flow.....	505
	References.....	506
Review on Harmonic Diffeomorphisms Between Complete Noncompact Surfaces		
	<i>Tom Y. H. Wan</i>	509
1	Introduction.....	509
2	Harmonic Map Theory of Universal Teichmüller Space.....	510

3	Asymptotic Behavior of Open Harmonic Embedding From the Complex Plane Into Hyperbolic Plane.....	512
	References.....	515
Compactifications of Complete Riemannian Manifolds and Their Applications		
	<i>Xiaodong Wang</i>	517
1	Introduction.....	517
2	The Geometric Compactification.....	518
3	The Martin Compactification.....	519
4	The Busemann Boundary.....	520
5	A Comparison Theorem.....	524
	References.....	528
Some Aspects of Weil-Petersson Geometry of Teichmüller Spaces		
	<i>Sumio Yamada</i>	531
1	Introduction.....	531
2	Harmonic Maps into \overline{T} and an Application.....	533
3	Finite Rank Properties of \overline{T}	537
4	Coxeter-Tits Construction.....	539
5	Weil-Petersson Geodesic Completeness.....	541
6	Weil-Petersson Geometry of the Universal Teichmüller Space.....	541
7	Embeddings of the Coxeter Complex into UT	543
8	Summary and Open Problems.....	544
	References.....	545