

Contents

Preface	xv
1 P. Bézier: How a Simple System Was Born	1
2 Introductory Material	13
2.1 Points and Vectors	13
2.2 Affine Maps	17
2.3 Linear Interpolation	19
2.4 Piecewise Linear Interpolation	22
2.5 Menelaos' Theorem	22
2.6 Function Spaces	24
2.7 Problems	26
3 The de Casteljaou Algorithm	27
3.1 Parabolas	27
3.2 The de Casteljaou Algorithm	29
3.3 Some Properties of Bézier Curves	30
3.4 Implementation	33
3.5 Problems	35
4 The Bernstein Form of a Bézier Curve	37
4.1 Bernstein Polynomials	37

4.2	Properties of Bézier Curves	40
4.3	The Derivative of a Bézier Curve	42
4.4	Higher Order Derivatives	44
4.5	Derivatives and the de Casteljaou Algorithm	45
4.6	The Matrix Form of a Bézier Curve	46
4.7	Implementation	47
4.8	Problems	49
5	Bézier Curve Topics	51
5.1	Degree Elevation	51
5.2	Repeated Degree Elevation	53
5.3	The Variation Diminishing Property	54
5.4	Degree Reduction	54
5.5	Nonparametric Curves	56
5.6	Cross Plots	57
5.7	Integrals	58
5.8	The Bézier Form of a Bézier Curve	60
5.9	The Barycentric Form of a Bézier Curve	61
5.10	The Weierstrass Approximation Theorem	63
5.11	Formulas for Bernstein Polynomials	64
5.12	Implementation	65
5.13	Problems	66
6	Polynomial Interpolation	67
6.1	Aitken's Algorithm	67
6.2	Lagrange Polynomials	71
6.3	The Vandermonde Approach	72
6.4	Limits of Lagrange Interpolation	73
6.5	Cubic Hermite Interpolation	75
6.6	Quintic Hermite Interpolation	79
6.7	The Newton Form and Forward Differencing	80
6.8	Implementation	82
6.9	Problems	83
7	Spline Curves in Bézier Form	85
7.1	Global and Local Parameters	85
7.2	Subdivision I	87
7.3	Subdivision II	89
7.4	Smoothness Conditions	91
7.5	C^1 Continuity	93
7.6	C^2 Continuity	94
7.7	Finding a C^1 Parametrization	95

7.8	C^1 Quadratic B-spline Curves	97
7.9	C^2 Cubic B-spline Curves	101
7.10	Parametrizations	103
7.11	Design and Inverse Design	104
7.12	Implementation	107
7.13	Problems	109
8	Piecewise Cubic Interpolation	111
8.1	C^1 Piecewise Cubic Hermite Interpolation	111
8.2	C^1 Piecewise Cubic Interpolation I	113
8.3	C^1 Piecewise Cubic Interpolation II	116
8.4	Point-Normal Interpolation	119
8.5	Problems	120
9	Cubic Spline Interpolation	121
9.1	The B-spline Form	121
9.2	The Hermite Form	125
9.3	End Conditions	127
9.4	The Parametrization	130
9.5	The Minimum Property	135
9.6	Implementation	139
9.7	Problems	145
10	B-splines	147
10.1	Motivation	148
10.2	Knot Insertion	149
10.3	The de Boor Algorithm	154
10.4	Smoothness of B-spline Curves	158
10.5	The B-spline Basis	158
10.6	Two Recursion Formulas	161
10.7	Repeated Knot Insertion	164
10.8	Additional Material	166
10.9	B-spline Basics	168
10.10	Implementation	170
10.11	Problems	171
11	W. Boehm: Differential Geometry I	173
11.1	Parametric Curves and Arc Length	173
11.2	The Frenet Frame	175
11.3	Moving the Frame	176
11.4	The Osculating Circle	178
11.5	Nonparametric Curves	181

11.6	Composite Curves	182
12	Geometric Continuity I	185
12.1	Motivation	185
12.2	A Characterization of G^2 Curves	186
12.3	Nu-splines	188
12.4	G^2 Piecewise Bézier Curves	191
12.5	Direct G^2 Cubic Splines	194
12.6	Implementation	195
12.7	Problems	197
13	Geometric Continuity II	199
13.1	Gamma-splines	199
13.2	Local Basis Functions for G^2 Splines	202
13.3	Beta-splines	205
13.4	Higher Order Geometric Continuity	207
13.5	Implementation	209
13.6	Problems	211
14	Conic Sections	213
14.1	Projective Maps of the Real Line	213
14.2	Conics as Rational Quadratics	217
14.3	A de Casteljau Algorithm	222
14.4	Derivatives	223
14.5	The Implicit Form	224
14.6	Two Classic Problems	226
14.7	Classification	227
14.8	Problems	229
15	Rational Bézier and B-spline Curves	231
15.1	Rational Bézier Curves	231
15.2	The de Casteljau Algorithm	234
15.3	Derivatives	235
15.4	Osculatory Interpolation	237
15.5	Reparametrization and Degree Elevation	237
15.6	Rational Cubic B-spline Curves	240
15.7	Interpolation with Rational Cubics	242
15.8	Rational B-splines of Arbitrary Degree	243
15.9	Implementation	244
15.10	Problems	247
16	Tensor Product Bézier Surfaces	249
16.1	Bilinear Interpolation	249

16.2	The Direct de Casteljau Algorithm	252
16.3	The Tensor Product Approach	254
16.4	Properties	257
16.5	Degree Elevation	258
16.6	Derivatives	259
16.7	Normal Vectors	261
16.8	Twists	263
16.9	The Matrix Form of a Bézier Patch	265
16.10	Nonparametric Patches	266
16.11	Implementation	267
16.12	Problems	269
17	Composite Surfaces and Spline Interpolation	271
17.1	Smoothness and Subdivision	271
17.2	Bicubic B-spline Surfaces	273
17.3	Twist Estimation	276
17.4	Tensor Product Interpolants	281
17.5	The Parametrization	284
17.6	Bicubic Hermite Patches	286
17.7	Rational Bézier and B-spline Surfaces	288
17.8	Volume Deformations	292
17.9	Trimmed Surfaces	295
17.10	Implementation	296
17.11	Problems	300
18	Bézier Triangles	303
18.1	Barycentric Coordinates and Linear Interpolation	304
18.2	The de Casteljau Algorithm	306
18.3	Bernstein Polynomials	308
18.4	Derivatives	310
18.5	Subdivision	315
18.6	Differentiability	317
18.7	Degree Elevation	320
18.8	Nonparametric Patches	322
18.9	Rational Bézier Triangles	324
18.10	Quadrics	326
18.11	Problems	329
19	Geometric Continuity for Surfaces	331
19.1	Introduction	331
19.2	Triangle-Triangle	332
19.3	Rectangle-Rectangle	336

19.4	Rectangle-Triangle	337
19.5	“Filling in” Rectangular Patches	338
19.6	“Filling in” Triangular Patches	339
19.7	Theoretical Aspects	339
19.8	Problems	340
20	Coons Patches	341
20.1	Ruled Surfaces	342
20.2	Coons Patches: Bilinearly Blended	343
20.3	Coons Patches: Partially Bicubically Blended	346
20.4	Coons Patches: Bicubically Blended	348
20.5	Piecewise Coons Surfaces	349
20.6	Problems	350
21	Coons Patches: Additional Material	351
21.1	Compatibility	351
21.2	Control Nets from Coons Patches	354
21.3	Translational Surfaces	356
21.4	Gordon Surfaces	357
21.5	Boolean Sums	359
21.6	Triangular Coons Patches	361
21.7	Implementation	363
21.8	Problems	364
22	W. Boehm: Differential Geometry II	367
22.1	Parametric Surfaces and Arc Element	367
22.2	The Local Frame	370
22.3	The Curvature of a Surface Curve	371
22.4	Meusnier’s Theorem	372
22.5	Lines of Curvature	373
22.6	Gaussian and Mean Curvature	375
22.7	Euler’s Theorem	376
22.8	Dupin’s Indicatrix	377
22.9	Asymptotic Lines and Conjugate Directions	378
22.10	Ruled Surfaces and Developables	379
22.11	Nonparametric Surfaces	381
22.12	Composite Surfaces	382
23	Interrogation and Smoothing	385
23.1	Use of Curvature Plots	385
23.2	Curve and Surface Smoothing	386
23.3	Surface Interrogation	388
23.4	Implementation	393

Contents	xiii
24 Evaluation of Some Methods	397
24.1 Bézier Curves or B-spline Curves?	397
24.2 Spline Curves or B-spline Curves?	397
24.3 The Monomial or the Bézier Form?	398
24.4 The B-spline or the Hermite Form?	399
24.5 Triangular or Rectangular Patches?	400
25 Quick Reference of Curve and Surface Terms	403
List of Programs	409
Notation	411
Bibliography	413
Index	435