

Contents

Chapter 1. Introduction	1
1.1. The background	1
1.2. The main results	2
1.3. The structure of the paper	5
Acknowledgments	6
Chapter 2. DG categories and dg quotients	7
2.1. DG categories and dg functors	7
2.2. One-point (co)extensions and singular equivalences with levels	10
Chapter 3. The dg singularity category and acyclic complexes	13
Chapter 4. Quivers and Leavitt path algebras	17
Chapter 5. An introduction to B_∞ -algebras	21
5.1. A_∞ -algebras and morphisms	21
5.2. B_∞ -algebras and morphisms	23
5.3. A duality theorem on B_∞ -algebras	25
5.4. Brace B_∞ -algebras	30
5.5. Gerstenhaber algebras	33
Chapter 6. The Hochschild cochain complexes	35
6.1. The Hochschild cochain complex of a dg category	35
6.2. The relative bar resolutions	37
Chapter 7. A homotopy deformation retract and the homotopy transfer theorem	41
7.1. A construction for homotopy deformation retracts	41
7.2. A homotopy deformation retract for the Leavitt path algebra	43
7.3. The homotopy transfer theorem for dg algebras	45
Chapter 8. The singular Hochschild cochain complexes	49
8.1. The left and right singular Hochschild cochain complexes	49
8.2. The B_∞ -algebra structures on the singular Hochschild cochain complexes	52
8.3. The relative singular Hochschild cochain complexes	60
Chapter 9. B_∞ -quasi-isomorphisms induced by one-point (co)extensions and bimodules	63
9.1. Invariance under one-point (co)extensions	63
9.2. B_∞ -quasi-isomorphisms induced by a bimodule	67
9.3. A non-standard resolution and liftings	68

9.4. A triangular matrix algebra and colimits	73
Chapter 10. Algebras with radical square zero and the combinatorial B_∞ -algebra	79
10.1. A combinatorial description of the singular Hochschild cochain complex	79
10.2. The combinatorial B_∞ -algebra	81
Chapter 11. The Leavitt B_∞ -algebra as an intermediate object	89
11.1. An explicit complex	89
11.2. The Leavitt B_∞ -algebra	90
11.3. A recursive formula for the brace operation	95
Chapter 12. An A_∞ -quasi-isomorphism for the Leavitt path algebra	97
12.1. An induced homotopy deformation retract	97
12.2. An explicit A_∞ -quasi-isomorphism between dg algebras	101
12.3. The A_∞ -quasi-isomorphism via the brace operation	103
Chapter 13. Verifying the B_∞ -morphism	105
Chapter 14. Keller's conjecture and the main results	107
Bibliography	111
Index	115