

MATHEMATICS FOR ECONOMISTS

An Introductory Textbook

Fifth Edition

Malcolm Pemberton and Nicholas Rau

MANCHESTER UNIVERSITY PRESS

Contents

Preface	xi
Dependence of Chapters	xvi
Answers and Solutions	xvii
The Greek Alphabet	xviii
1 LINEAR EQUATIONS	1
1.1 Straight line graphs	1
1.2 An economic application: supply and demand	8
1.3 Simultaneous equations	10
1.4 Input-output analysis	17
Problems on Chapter 1	19
2 LINEAR INEQUALITIES	21
2.1 Inequalities	21
2.2 Economic applications	25
2.3 Linear programming	29
Problems on Chapter 2	34
3 SETS AND FUNCTIONS	36
3.1 Sets	36
3.2 Real numbers	41
3.3 Functions	47
3.4 Mappings	55
Problems on Chapter 3	57
4 QUADRATICS, INDICES AND LOGARITHMS	59
4.1 Quadratic functions and equations	59
4.2 Maximising and minimising quadratic functions	66
4.3 Indices	68
4.4 Logarithms	73
Problems on Chapter 4	77

5	SEQUENCES, SERIES AND LIMITS	79
5.1	Sequences	79
5.2	Series	84
5.3	Geometric progressions in economics	88
5.4	Limits and continuity	93
	Problems on Chapter 5	100
6	INTRODUCTION TO DIFFERENTIATION	102
6.1	The derivative	103
6.2	Linear approximations and differentiability	110
6.3	Two useful rules	114
6.4	Derivatives in economics	117
	Problems on Chapter 6	119
7	METHODS OF DIFFERENTIATION	121
7.1	The product and quotient rules	121
7.2	The composite function rule	124
7.3	Monotonic functions	128
7.4	Inverse functions	133
	Problems on Chapter 7	137
	Appendix to Chapter 7	139
8	MAXIMA AND MINIMA	140
8.1	Critical points	140
8.2	The second derivative	144
8.3	Optimisation	148
8.4	Convexity and concavity	156
	Problems on Chapter 8	163
9	EXPONENTIAL AND LOGARITHMIC FUNCTIONS	165
9.1	The exponential function	165
9.2	Natural logarithms	170
9.3	Time in economics	176
	Problems on Chapter 9	179
	Appendix to Chapter 9	181
10	APPROXIMATIONS	183
10.1	Linear approximations and Newton's method	184
10.2	The mean value theorem	187
10.3	Quadratic approximations and Taylor's theorem	191
10.4	Taylor and Maclaurin series	195
	Problems on Chapter 10	199
	Appendix to Chapter 10	200

11 MATRIX ALGEBRA	203
11.1 Vectors	203
11.2 Matrices	209
11.3 Matrix multiplication	216
11.4 Square matrices	220
Problems on Chapter 11	222
12 SYSTEMS OF LINEAR EQUATIONS	225
12.1 Echelon matrices	225
12.2 More on Gaussian elimination	229
12.3 Inverting a matrix	235
12.4 Linear independence and rank	241
Problems on Chapter 12	246
13 DETERMINANTS AND QUADRATIC FORMS	248
13.1 Determinants	249
13.2 Transposition	255
13.3 Inner products	258
13.4 Quadratic forms and symmetric matrices	262
Problems on Chapter 13	269
Appendix to Chapter 13	272
14 FUNCTIONS OF SEVERAL VARIABLES	273
14.1 Partial derivatives	274
14.2 Approximations and the chain rule	280
14.3 An economic application: production functions	286
14.4 Homogeneous functions	290
Problems on Chapter 14	295
Appendix to Chapter 14	298
15 IMPLICIT RELATIONS	301
15.1 Implicit differentiation	301
15.2 Comparative statics	309
15.3 Generalising to higher dimensions	315
Problems on Chapter 15	319
Appendix to Chapter 15	323
16 OPTIMISATION WITH SEVERAL VARIABLES	325
16.1 Critical points and their classification	325
16.2 Global optima, concavity and convexity	333
16.3 Non-negativity constraints	341
Problems on Chapter 16	344
Appendix to Chapter 16	346

17 PRINCIPLES OF CONSTRAINED OPTIMISATION	348
17.1 Lagrange multipliers	348
17.2 Extensions and warnings	355
17.3 Economic applications	359
17.4 Quasi-concave functions	368
Problems on Chapter 17	374
18 FURTHER TOPICS IN CONSTRAINED OPTIMISATION	378
18.1 The meaning of the multipliers	379
18.2 Envelope theorems	382
18.3 Non-negativity constraints again	388
18.4 Inequality constraints	392
Problems on Chapter 18	400
19 INTEGRATION	403
19.1 Areas and integrals	403
19.2 Rules of integration	410
19.3 Integration in economics	416
19.4 Numerical integration	419
Problems on Chapter 19	426
Appendix to Chapter 19	429
20 ASPECTS OF INTEGRAL CALCULUS	431
20.1 Methods of integration	431
20.2 Infinite integrals	437
20.3 Differentiation under the integral sign	442
20.4 Double integrals	447
Problems on Chapter 20	454
21 PROBABILITY	456
21.1 Events and their probabilities	456
21.2 Conditional probability and independence	462
21.3 Random variables	469
21.4 The binomial, Poisson and exponential distributions	476
Problems on Chapter 21	479
Appendix to Chapter 21	481
22 EXPECTATION	483
22.1 Expected value	483
22.2 The variance and higher moments	488
22.3 Two or more random variables	494
22.4 Random samples and limit theorems	504
Problems on Chapter 22	510
Appendix to Chapter 22	513

23 INTRODUCTION TO DYNAMICS	515
23.1 Differential equations	515
23.2 Linear equations with constant coefficients	520
23.3 Harder first-order equations	526
23.4 Difference equations	532
Problems on Chapter 23	540
24 THE CIRCULAR FUNCTIONS	542
24.1 Cycles, circles and trigonometry	542
24.2 Extending the definitions	548
24.3 Calculus with circular functions	555
24.4 Polar coordinates	561
Problems on Chapter 24	564
25 COMPLEX NUMBERS	566
25.1 The complex number system	566
25.2 The trigonometric form	571
25.3 Complex exponentials and polynomials	575
Problems on Chapter 25	581
26 FURTHER DYNAMICS	583
26.1 Second-order differential equations	583
26.2 Qualitative behaviour	592
26.3 Second-order difference equations	598
Problems on Chapter 26	606
Appendix to Chapter 26	609
27 EIGENVALUES AND EIGENVECTORS	611
27.1 Diagonalisable matrices	611
27.2 The characteristic polynomial	617
27.3 Eigenvalues of symmetric matrices	624
Problems on Chapter 27	628
28 DYNAMIC SYSTEMS	631
28.1 Systems of difference equations	631
28.2 Systems of differential equations	639
28.3 Qualitative behaviour	644
28.4 Non-linear systems	656
Problems on Chapter 28	663
Appendix to Chapter 28	665
29 DYNAMIC OPTIMISATION IN DISCRETE TIME	666
29.1 The basic problem	667
29.2 An application: consumption over time	670
29.3 Variants of the basic problem	675

29.4 Dynamic programming	679
Problems on Chapter 29	685
Appendix to Chapter 29	688
30 DYNAMIC OPTIMISATION IN CONTINUOUS TIME	690
30.1 The basic problem and its variants	690
30.2 The maximum principle	695
30.3 Two applications to resource economics	700
30.4 Problems with an infinite horizon	707
Problems on Chapter 30	710
Appendix to Chapter 30	714
31 INTRODUCTION TO ANALYSIS	719
31.1 Rigour	719
31.2 More on the real number system	723
31.3 Sequences of real numbers	727
31.4 More on limits and continuity	732
Problems on Chapter 31	735
32 METRIC SPACES AND EXISTENCE THEOREMS	738
32.1 Metric spaces	739
32.2 Open, closed and compact sets	744
32.3 Continuous mappings	749
32.4 Fixed point theorems	754
Problems on Chapter 32	758
33 FURTHER LINEAR ALGEBRA	761
33.1 Vector spaces	761
33.2 Orthogonality and projections	769
33.3 Convex polyhedra	780
33.4 Theory of linear programming	786
Problems on Chapter 33	794
34 CONVEX ANALYSIS	796
34.1 More on open and closed sets	796
34.2 Separation of convex sets	802
34.3 The fundamental theorems of welfare economics	809
34.4 More on convex functions	814
Problems on Chapter 34	820
Appendix to Chapter 34	823
Notes on Further Reading	825
Index	827