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REGULARIZED UNBALANCED OPTIMAL TRANSPORT
AS ENTROPY MINIMIZATION
WITH RESPECT TO BRANCHING BROWNIAN MOTION

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