

Contents

1	Introduction	1
1.1	Morse Homology with Integer Coefficients	1
1.2	Extensions and Applications	3
1.3	Summary of Main Results	4
2	The Morse Complex with Local Coefficients	11
2.1	Local Coefficients	11
2.2	Path Components of Compactified Moduli Spaces	13
2.3	Twisting the Morse-Smale-Witten Boundary Operator	16
2.4	Examples	18
2.5	Computation of $H_0((C_*(f; G), \partial_*^G))$	23
2.6	The Morse Eilenberg Theorem	24
2.6.1	The Morse-Smale-Witten Chain Complex on a Covering Space	25
2.6.2	The Morse Eilenberg Theorem	29
3	The Homology Determined by the Isomorphism Class of G	37
3.1	A Chain Map	37
3.2	A Chain Homotopy	42
3.3	An Invariance Theorem	49
4	Singular and CW-Homology with Local Coefficients	51
4.1	Singular Homology with Local Coefficients	52
4.2	Regular CW-Complexes	53
4.3	Unstable Manifolds and Regular CW-Structures	58
4.4	A Morse-Smale Function that Determines a Regular CW-Structure	61
4.4.1	Outline of the Proof	62
4.4.2	The Construction on a 2-Simplex	64
4.4.3	The Construction for Adjacent 2-Cells	68
4.4.4	Proof of Theorem 4.12	71
4.5	Local Coefficient Systems of R -Modules and the Euler Number	83

5	Twisted Morse Cohomology and Lichnerowicz Cohomology	89
5.1	Twisted Morse Cohomology	89
5.2	Lichnerowicz Cohomology and LCS Manifolds	92
5.3	Mapping Differential Forms to Morse-Smale-Witten Cochains	96
5.4	Relationship to Sheaf Cohomology	104
6	Applications and Computations	107
6.1	Parallel 1-Forms and Lichnerowicz Cohomology Computations	107
6.2	H-Spaces.....	118
6.3	Novikov Homology	124
6.3.1	A Covering Space Associated to a 1-Form	125
6.3.2	Novikov Rings	128
6.3.3	A Local Coefficient System of Rank One Nov-Modules	128
6.3.4	Novikov Homology	130
6.3.5	Novikov Numbers	135
6.3.6	Novikov Inequalities.....	136
6.3.7	Novikov Numbers and Twisted Morse Homology	137
	References	147
	Index	153