

Contents

Chapter 1. Introduction	1
Chapter 2. Main ideas and strategy	9
2.1. Main steps	9
2.2. The main bootstrap argument	13
Chapter 3. Linear Spectral Theory	15
3.1. Generalized eigenfunctions and distorted Fourier transform	15
3.2. Bounds on $\psi(x, k)$	16
3.3. Expansion of ψ	19
Chapter 4. Preliminary bounds: Linear estimates and high frequencies	25
4.1. Linear Estimates	25
4.2. Sobolev estimates	26
4.3. Weighted estimates for high frequencies	27
Chapter 5. Analysis of the NSD I: structure of the leading order	35
5.1. Expansion of μ and leading order nonlinear terms	35
5.2. Structure and properties of ν_1	36
5.3. Proof of Lemma 5.2	39
5.4. Proof of Proposition 5.1	44
Chapter 6. Bilinear estimates for the leading order of the NSD	49
6.1. Bilinear estimates for μ_1	49
6.2. Bilinear operators supported on thin annuli	52
6.3. Proof of Theorem 6.1	54
6.4. Proof of Theorem 6.2	61
6.5. Proof of Lemma 6.4	62
Chapter 7. Weighted estimates for leading order terms	67
7.1. Proof of (7.3)	68
7.2. Proof of (7.4)	74
7.3. Estimates for $\mathcal{N}_{2,S}$	79
Chapter 8. Analysis of the NSD II: Lower order terms	85
8.1. Analysis of μ_2 : Structure	85
8.2. Analysis of μ_2 : Bilinear estimates	91
8.3. Analysis of μ_3 : structure and bilinear estimates	97
Chapter 9. Weighted estimates for lower order terms	101
9.1. Estimates for $\mathcal{D}_2$	101
9.2. Nonlinear Estimates for $\mathcal{D}_0$	104

Bibliography

105