

# Contents

Chapter 1. Introduction and Background Review	1
1.1. Gross-Ruan-Joyce picture of SYZ fibrations	3
1.2. Generalised Gibbons-Hawking ansatz	9
1.3. Ooguri-Vafa metric	15
1.4. Synopsis of the new metrics	18
Acknowledgment	22
Chapter 2. Taub-NUT Type Metrics on $\mathbb{C}^3$	23
2.1. First order asymptotic metric near infinity	24
2.2. Metric behaviour away from the discriminant locus	28
2.3. Structure near discriminant locus	30
2.4. Complex geometric perspective	33
2.5. Algebraic geometric perspective	37
2.6. Surgery on the ansatz	39
2.7. Hein's package and weighted Sobolev inequality	40
2.8. Harmonic analysis	43
2.9. Perturbation into a Calabi-Yau metric	49
2.10. Uniqueness and moduli	52
2.11. Exotic metrics: past and future	53
Chapter 3. The Positive Vertex	61
3.1. First order approximate metric	61
3.2. Asymptotes for the first order ansatz	64
3.3. Complex geometric perspective	67
3.4. Weighted Hölder norms and initial error estimates	71
3.5. Harmonic analysis I: periodic Euclidean region	73
3.6. Perturbation in the Euclidean region	75
3.7. Glue in the Taub-NUT type metric on $\mathbb{C}^3$	77
3.8. Harmonic analysis II: perturbation to Calabi-Yau metric	80
3.9. Ooguri-Vafa type metric on the positive vertex	81
3.10. Incompleteness and running coupling	83
Chapter 4. The Negative Vertex	89
4.1. First order approximate metric	90
4.2. Asymptotic for the first order ansatz I	96
4.3. Asymptotic for the first order ansatz II	97
4.4. Structure near the singular locus I	99
4.5. Structure near the singular locus II	103
4.6. Complex geometric perspective	105
4.7. Weighted Hölder norms and initial error estimate	115

4.8. Harmonic analysis I: periodic Euclidean region	117
4.9. Harmonic analysis II: Neighbourhood of $S$	118
4.10. Harmonic analysis III: perturbation to Calabi-Yau metric	120
4.11. Ooguri-Vafa type metrics on the negative vertex	121
4.12. Special Lagrangian geometry	121
4.13. Incompleteness and running coupling	123
Bibliography	125