

CONTENTS

CHAPTER I

INFINITE SERIES, PRODUCTS, AND INTEGRALS

1.1. Uniform convergence of series	2
1.2. Series of complex terms. Power series	8
1.3. Series which are not uniformly convergent	11
1.4. Infinite products	13
1.5. Infinite integrals	19
1.6. Double series	27
1.7. Integration of series	36
1.8. Repeated integrals. The Gamma-function	48
1.88. Differentiation of integrals	59

CHAPTER II

ANALYTIC FUNCTIONS

2.1. Functions of a complex variable	64
2.2. The complex differential calculus	70
2.3. Complex integration. Cauchy's theorem	71
2.4. Cauchy's integral. Taylor's series	80
2.5. Cauchy's inequality. Liouville's theorem	84
2.6. The zeros of an analytic function	87
2.7. Laurent series. Singularities	89
2.8. Series and integrals of analytic functions	95
2.9. Remark on Laurent Series	101

CHAPTER III

RESIDUES, CONTOUR INTEGRATION, ZEROS

3.1. Residues. Contour integration	102
3.2. Meromorphic functions. Integral functions	110
3.3. Summation of certain series	114
3.4. Poles and zeros of a meromorphic function	115
3.5. The modulus, and real and imaginary parts, of an analytic function	119
3.6. Poisson's integral. Jensen's theorem	124
3.7. Carleman's theorem	130
3.8. A theorem of Littlewood	132

CHAPTER IV

ANALYTIC CONTINUATION

4.1. General theory	138
4.2. Singularities	143
4.3. Riemann surfaces.	148

4.4. Functions defined by integrals. The Gamma-function. The Zeta-function	147
4.5. The principle of reflection	155
4.6. Hadamard's multiplication theorem	157
4.7. Functions with natural boundaries	159

CHAPTER V

THE MAXIMUM-MODULUS THEOREM

5.1. The maximum-modulus theorem	165
5.2. Schwarz's theorem. Vitali's theorem. Montel's theorem	168
5.3. Hadamard's three-circles theorem	172
5.4. Mean values of $ f(z) $	173
5.5. The Borel-Carathéodory inequality	174
5.6. The Phragmén-Lindelöf theorems	176
5.7. The Phragmén-Lindelöf function $h(\theta)$	181
5.8. Applications	185

CHAPTER VI

CONFORMAL REPRESENTATION

6.1. General theory	188
6.2. Linear transformations	190
6.3. Various transformations	195
6.4. Simple (<i>schlicht</i>) functions	198
6.5. Application of the principle of reflection	203
6.6. Representation of a polygon on a half-plane	205
6.7. General existence theorems	207
6.8. Further properties of simple functions	209

CHAPTER VII

POWER SERIES WITH A FINITE RADIUS OF CONVERGENCE

7.1. The circle of convergence	213
7.2. Position of the singularities	214
7.3. Convergence of the series and regularity of the function	217
7.4. Over-convergence. Gap theorems	220
7.5. Asymptotic behaviour near the circle of convergence	224
7.6. Abel's theorem and its converse	229
7.7. Partial sums of a power series	235
7.8. The zeros of partial sums	238

CHAPTER VIII

INTEGRAL FUNCTIONS

8.1. Factorization of integral functions	246
8.2. Functions of finite order	248

8.3. The coefficients in the power series	253
8.4. Examples	255
8.5. The derived function	265
8.6. Functions with real zeros only	268
8.7. The minimum modulus	273
8.8. The a -points of an integral function. Picard's theorem	277
8.9. Meromorphic functions	284 b

CHAPTER IX

DIRICHLET SERIES

9.1. Introduction. Convergence. Absolute convergence	289
9.2. Convergence of the series and regularity of the function	294
9.3. Asymptotic behaviour	295
9.4. Functions of finite order	298
9.5. The mean-value formula and half-plane	303
9.6. The uniqueness theorem. Zeros	309
9.7. Representation of functions by Dirichlet series	313

CHAPTER X

THE THEORY OF MEASURE AND THE LEBESGUE
INTEGRAL

10.1. Riemann integration	318
10.2. Sets of points. Measure	319
10.3. Measurable functions	330
10.4. The Lebesgue integral of a bounded function	332
10.5. Bounded convergence	337
10.6. Comparison between Riemann and Lebesgue integrals	339
10.7. The Lebesgue integral of an unbounded function	341
10.8. General convergence theorems	345
10.9. Integrals over an infinite range	347

CHAPTER XI

DIFFERENTIATION AND INTEGRATION

11.1. Introduction	349
11.2. Differentiation throughout an interval. Non-differentiable functions	350
11.3. The four derivates of a function	354
11.4. Functions of bounded variation	355
11.5. Integrals	359
11.6. The Lebesgue set	362
11.7. Absolutely continuous functions	364
11.8. Integration of a differential coefficient	367

CHAPTER XII

FURTHER THEOREMS ON LEBESGUE INTEGRATION

12.1. Integration by parts	375
12.2. Approximation to an integrable function. Change of the independent variable	376
12.3. The second mean-value theorem	379
12.4. The Lebesgue class L^p	381
12.5. Mean convergence	386
12.6. Repeated integrals	390

CHAPTER XIII

FOURIER SERIES

13.1. Trigonometrical series and Fourier series	399
13.2. Dirichlet's integral. Convergence tests	402
13.3. Summation by arithmetic means	411
13.4. Continuous functions with divergent Fourier series	416
13.5. Integration of Fourier series. Parseval's theorem	419
13.6. Functions of the class L^2 . Bessel's inequality. The Riesz- Fischer theorem	422
13.7. Properties of Fourier coefficients	425
13.8. Uniqueness of trigonometrical series	427
13.9. Fourier integrals	432
BIBLIOGRAPHY	445
GENERAL INDEX	453