

Contents

1	The Beginning	1
1.1	The Congruence \mathcal{H}	1
1.1.1	Basics	1
1.1.2	Commutative Group Coextensions	3
1.2	Construction	4
1.2.1	Schreier's Method	4
1.2.2	Split Coextensions	5
1.2.3	Enter Cohomology	7
1.2.4	Finite Semigroups	7
2	Beck Cohomology	9
2.1	General Beck Cohomology	9
2.1.1	Simple Cohomology	9
2.1.2	Abelian Group Objects	10
2.1.3	Objects Over S	11
2.1.4	Beck Cohomology	12
2.1.5	Main Properties	13
2.1.6	Beck Extensions	13
2.2	Commutative Semigroups	14
2.2.1	Commutative Semigroups Over S	14
2.2.2	Abelian Group Objects Over S	15
2.2.3	Beck Extensions of S	17
2.3	Beck Cohomology of Commutative Semigroups	18
2.3.1	The 'Free Commutative Semigroup' Adjunction	18
2.3.2	The 'Free Commutative Semigroup' Comonad	19
2.3.3	Cochains	20
2.3.4	Cohomology	22
2.3.5	Properties	25
3	Symmetric Cohomology	27
3.1	Definition	27
3.1.1	Cochains	28

3.1.2	Symmetric Cochains	28
3.1.3	Symmetric Cohomology	29
3.1.4	An Example	30
3.2	Comparison with Beck Cohomology	32
3.2.1	Dimension 1	32
3.2.2	Dimension 2	32
3.2.3	Dimensions 3 and 4	37
3.3	Main Properties	38
3.4	Normalization	38
3.4.1	Dimension 2	39
3.4.2	Dimension 3	41
4	Calvo-Cegarra Cohomology	45
4.1	Small Categories	45
4.2	Cohomology of Simplicial Sets	46
4.2.1	Definition	46
4.2.2	Cochains	47
4.2.3	The Classifying Simplicial Set	48
4.3	Cohomology of Commutative Semigroups	49
4.3.1	The Double Classifying Simplicial Set	49
4.3.2	Cochains	51
4.4	Extended Cochains	52
4.4.1	Definition	52
4.4.2	Comparison with Symmetric Cohomology	54
4.4.3	An Example	55
4.5	Properties	57
5	The Third Cohomology Group	59
5.1	Groupoids	59
5.1.1	Groupoids	59
5.1.2	Monoidal Groupoids	60
5.1.3	Reduction	63
5.1.4	The Base	64
5.2	Symmetric 3-Cocycles	66
5.2.1	Cocycle Objects	66
5.2.2	Morphisms	67
5.3	Classification	73
5.3.1	Isomorphisms	73
5.3.2	Equivalence	75
5.3.3	Lone Cocycles	76
5.4	Braided Groupoids	78
5.4.1	Definition	78
5.4.2	Reduction	80
5.4.3	The Base	81
5.4.4	Extended Cocycle Objects	82
5.4.5	Classification	83

6	The Overpath Method	87
6.1	Paths and Overpaths	87
6.1.1	Free Commutative Monoids	87
6.1.2	Congruences	88
6.1.3	Paths	89
6.1.4	Overpaths	91
6.2	Main Result	91
6.2.1	Minimal Cocycles	92
6.2.2	Main Result	93
6.2.3	Examples	94
6.2.4	Semigroups with One Relator	96
6.3	Other Results	98
6.3.1	Branching	98
6.3.2	Relations	100
6.3.3	Partially Free Semigroups	103
6.3.4	Nilmonoids	104
6.3.5	Semigroups with Zero Cohomology	105
7	Symmetric Chains	109
7.1	Symmetric Mappings	109
7.1.1	Symmetry	109
7.1.2	Bases	110
7.2	Chain Groups	113
7.2.1	Definition	113
7.2.2	Properties	114
7.2.3	Symmetric n -chains	115
7.3	Chain Functors	117
7.3.1	Thin Chain Functors	117
7.3.2	General Chain Functors	119
7.4	Semiconstant Functors	121
7.4.1	Definition	122
7.4.2	Chain Groups	123
7.4.3	Properties	124
7.4.4	Homology	124
7.4.5	Cohomology	126
8	Inheritance	129
8.1	The Universal Coboundary	129
8.1.1	Symmetry Properties	129
8.1.2	The Universal Coboundary	130
8.1.3	The Group \mathbb{D}	131
8.2	One Equality Between Variables	134
8.3	Results	135
8.3.1	Method	136
8.3.2	Order 5	136
8.3.3	Other Orders	137

9	Appendixes	139
9.1	Extensions	139
9.1.1	Group Extensions	139
9.1.2	Rédei Extensions.....	141
9.1.3	The Leech Categories.....	141
9.1.4	Cosets	143
9.1.5	Group Coextensions	144
9.1.6	Congruences Contained in \mathcal{H}	146
9.1.7	Leech Coextensions	149
9.1.8	Leech Cohomology	151
9.2	Monads and Algebras	152
9.2.1	Adjunctions	152
9.2.2	Monads	153
9.2.3	Algebras	154
9.3	Simplicial Objects.....	155
9.3.1	Simplicial Sets	155
9.3.2	The Simplicial Category	156
9.3.3	The Classifying Simplicial Set	158
9.3.4	Cohomology	159
9.4	Monoidal Categories	159
9.4.1	Strict Monoidal Categories	160
9.4.2	General Monoidal Categories	160
9.4.3	Monoidal Functors.....	161
9.4.4	Braided Monoidal Categories	163
9.5	Modules	165
9.5.1	S -Modules.....	165
9.5.2	Quasiconstant Functors.....	166
9.5.3	Conclusions	166
	References	167
	Index	171