

# Contents

Chapter 1. Introduction	1
1.1. Avant-propos	1
1.2. Motivation	1
1.3. Main results	5
Acknowledgments	8
Chapter 2. Preliminaries	9
2.1. Basic notions	9
2.2. Further notions	14
Chapter 3. Local-to-Global Characterization	29
3.1. Main results	29
3.2. Weakly modular graphs: proofs	31
3.3. Helly and clique-Helly graphs: proofs	38
3.4. A note on meshed graphs	50
Chapter 4. Pre-Median Graphs	53
4.1. Main results	53
4.2. Prime pre-median graphs	55
4.3. Examples	61
4.4. $L_1$ -Weakly modular graphs	66
4.5. $C(G)$ is contractible	73
Chapter 5. Dual Polar Graphs	83
5.1. Main results	83
5.2. Proof of Theorem 5.2	85
5.3. Proof of Theorems 5.3 and 5.4	87
Chapter 6. Sweakly Modular Graphs	93
6.1. Main results	93
6.2. A lattice-theoretical characterization of swm-graphs	94
6.3. Boolean pairs and Boolean-gated sets	98
6.4. Barycentric graph $G^*$	100
6.5. Thickening $G^\Delta$	105
6.6. $\Delta$ -gates and geodesic extension property	107
6.7. Normal Boolean-gated paths	108
6.8. Euclidean buildings of type $C_n$	109
6.9. Application I: Biautomaticity of swm-groups	114
6.10. Application II: 2-approximation to 0-extension problem on swm-graphs	117

Chapter 7. Orthoscheme Complexes of Modular Lattices and Semilattices	121
7.1. Main results	122
7.2. Boolean and complemented modular lattices	124
7.3. Distributive and modular lattices	125
7.4. Proof of Theorem 7.2	128
7.5. Proof of Propositions 7.5 and 7.6	129
7.6. Proof of Proposition 7.4	130
Chapter 8. Orthoscheme Complexes of Swm-Graphs	133
8.1. Main results	133
8.2. Examples	134
8.3. Preliminary results	136
8.4. $l_1$ -metrization	140
8.5. $l_\infty$ -metrization	141
8.6. $l_2$ -metrization	143
8.7. $K(G)$ is contractible	143
Chapter 9. Metric Properties of Weakly Modular Graphs	145
9.1. Quadratic isoperimetric inequality	145
9.2. Hyperbolicity	146
9.3. BFS gives a distance-preserving ordering	149
9.4. Weakly modular complex	151
Bibliography	155